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(VOLUME II)

SPACECRAFT ATTITUDE CONTROL FOR EXTENDED MISSIONS

Noah C. New

Major USMC

May 1963

degree of Doctor of Science

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SPACECRAFT ATTITUDE CONTROL FOR EXTENDED MISSIONS

ABSTRACT

This thesis considers the problem of providing attitude control for a spacecraft engaged in an extended mission. As a basis for the choice of a suitable attitude control system the following requirements are applied.

Maximum reliability
Minimum ejection of mass
Minimum average power
Minimum system weight
Minimum peak power

An interplanetary mission of 400 days duration is adopted as a general guide for the problem, but most of the equations and comparisons are presented in parametric form. Extended missions imply that a momentum exchange type attitude control system be used to minimize ejection of fuel mass, and the thesis primarily considers only systems of this type. The thesis derives the equations of motion for a spacecraft equipped with eighteen different control systems. The control system chosen to best satisfy the five design requirements is a system consisting of four gyro-type controllers arranged in two pairs with each pair operating back-to-back to minimize control cross coupling torques. One pair of controllers provide roll torques, the other pair provides pitch torques, and all four controllers contribute yaw torques.

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Maximum reliability
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An interplanetary mission of 400 days duration is adopted as a general guide for the problem, but most of the equations and comparisons are presented in parametric form. Extended missions imply that a momentum exchange type attitude control system be used to minimize ejection of fuel mass, and the thesis primarily considers only systems of this type. The thesis derives the equations of motion for a spacecraft equipped with eighteen different control systems. The control system chosen to best satisfy the five design requirements is a system consisting of four gyro-type controllers arranged in two pairs with each pair operating back-to-back to minimize control cross coupling torques. One pair of controllers provide roll torques, and all four controllers

contribute yaw torques.

The stability and control analysis considers operation of the spacecraft in three modes.

Zero Input Mode
Rate Control Mode
Position Control Mode

Each of the modes are evaluated for roll motion by assuming negligible interaxial coupling, and the analysis includes operation of the controller gimbal angles to large angles.

Thesis Supervisor: Wallace E. Vander Velde

Title: Associate Professor of
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CHAPTER 6

CLOSING THE ATTITUDE CONTROL LOOP

6.1 Introduction

Previous chapters have defined the physical characteristics of the spacecraft, have chosen the four gyro (12-34-1234) system as being the most attractive control system, and have defined probable disturbances on the spacecraft. Before the mathematical models representing the spacecraft and control system can be combined into a single loop it is necessary to specify an attitude sensor and some type of torque motor which will receive signal information from the sensor and provide the proper angular rates and positions to the spin axes of the four controllers. The sequence of this chapter follows an order that first determines the dynamics of the individual controllers by methods similar to those of Chapter 2 which considered the complete spacecraft. Next, the motors and drive systems which drive the controllers are defined. Then, the sensor and related amplifiers are assumed together with the provision for tandem compensation, if required. Finally, all of the above components are combined into a single loop.

6.2 Response of the Controllers to Moments Applied by the Roll and Pitch Torque Generators

In a manner parallel to that of Section 2.3 the fundamental equation of motion of a controller consisting of a gyro case, a gimbal, and a rotor is

$$\left.\frac{dH}{dt}\right]_I = \sum M_{case}\Big]_I \qquad \text{(Eq 6.2.1)}$$

Since it is proposed to derive roll and pitch moments from the

control system by torquing the controllers about their vertical axis (γ axis), it is convenient to sum torques in the coordinate frame of the case (gu frame). Accordingly,

$$\sum M_{case}\Big]_{GU} = \frac{dH}{dt}\Big]_{GU} + W_{I,GU} \star H\Big]_{GU} \qquad \text{(Eq 6.2.2)}$$

where $H = H_r + H_g + H_c$

From Appendix D, Equation D. 3. 5,

$$W_{I,GU} = \begin{bmatrix} pC\gamma + qS\gamma \\ -pS\gamma + qC\gamma \\ r + \gamma \end{bmatrix} \qquad \text{(Eq 6.2.3)}$$

The approximation made in Section 2. 6 is that

$$H_r\Big]_{GIM} \approx \begin{bmatrix} 0 \\ J_p \dot{\beta} \\ 0 \end{bmatrix} \qquad \text{(Eq 6.2.4)}$$

Now, in the gu frame

$$H_r\Big]_{GU} = Q_{GU,GIM}\, H_r\Big]_{GIM} = \begin{bmatrix} 0 \\ J_p \dot{\beta} C\alpha \\ J_p \dot{\beta} S\alpha \end{bmatrix} \qquad \text{(Eq 6.2.5)}$$

Define J_{cgr} as the combined moment of inertia of the case,

gimbal and rotor.

$$J_{cgr} = J_c + J_g + J_r \qquad \text{(Eq 6. 2. 6)}$$

Combining the last two equations gives

$$H\rfloor_{GU} = \begin{bmatrix} 0 \\ J_p \dot{\beta} C\alpha \\ J_p \dot{\beta} S\alpha + J_{cgr} \dot{\gamma} \end{bmatrix} \qquad \text{(Eq 6. 2. 7)}$$

Differentiating with respect to time where it is assumed that angular momentum about the spin axis, $J_{P\beta}$, is constant gives

$$\dot{H}\rfloor_{GU} = \begin{bmatrix} 0 \\ -J_p \dot{\beta} S\alpha \dot{\alpha} \\ +J_p \dot{\beta} C\alpha \dot{\alpha} + J_{cgr} \ddot{\gamma} \end{bmatrix} \qquad \text{(Eq 6. 2. 8)}$$

Then substituting the appropriate values into Eq 6. 2. 2 gives for the Number 1 Controller the following.

$$\sum M_{case\ 1}\rfloor_{GU} = \begin{bmatrix} -(R + \dot{\gamma}) J_p \dot{\beta} C\alpha + (-pS\gamma + qC\gamma)(J_p \dot{\beta} S\alpha + J_{cgr} \dot{\gamma}) \\ -J_p \dot{\beta} S\alpha \dot{\alpha} - (pC\gamma + qS\gamma)(J_p \dot{\beta} S\alpha + J_{cgr} \dot{\gamma}) \\ +J_{cgr} \ddot{\gamma} + (\dot{\alpha} + pC\gamma + qS\gamma) J_p \dot{\beta} C\alpha \end{bmatrix}$$

(Eq 6. 2. 9)

The X and Y components of the torques in the last equation are normal to the single degree of freedom of the case, and therefore are transmitted through the bearing to the vehicle. The Z component is aligned with the bearing axis, and the rotation of the case must be controlled by a torque generator about the γ axis. The single Z component can be written as

$$M_{\gamma\ case\ 1} = J_{cgr} \ddot{\gamma} + J_p \dot{\beta} \left(\dot{\alpha} + pC\gamma + qS\gamma \right) C\alpha \qquad \text{(Eq 6. 2. 10)}$$

$M_{\gamma\ case\ 1}$ consists of a torque generated part, a load, and a viscous friction. Applying these to equation 6. 2. 10 gives for each of the controllers the following.

Controller 1 Let $F_1 = J_{cgr} \frac{d}{dt} + C$

$$M_{TG1} + M_{L1} = F_1 \dot{\gamma}_1 + J_{p_1} \dot{\beta}_1 (\dot{\alpha}_1 + pC\gamma_1 + qS\gamma_1) C\alpha_1$$

(Eq 6. 2. 11)

Controller 2 $(\gamma_2 = \gamma_1 + 180^{o})$

$$M_{TG2} + M_{L2} = F_2 \dot{\gamma}_2 + J_{p_2} \dot{\beta}_2 (\dot{\alpha}_2 - pC\gamma_2 - qS\gamma_2) C\alpha_2$$

(Eq 6. 2. 12)

Controller 3 $(\gamma_3 = \gamma_1 - 90^{o})$

$$M_{TG3} + M_{L3} = F_3 \dot{\gamma}_3 + J_{p_3} \dot{\beta}_3 (\dot{\alpha}_3 + pS\gamma_3 - qC\gamma_3) C\alpha_3$$

(Eq 6. 2. 13)

Controller 4 $(\gamma_4 = \gamma_1 + 90^{o})$

$$M_{TG4} + M_{L4} = F_4 \dot{\gamma}_4 + J_{p_4} \dot{\beta}_4 (\alpha_4 - pS\gamma_4 + qC\gamma_4) C\alpha_4$$

(Eq 6. 2. 14)

Combining Controllers 1 and 2 in accordance with the logic that $\gamma_2 = -\gamma_1$ and $\alpha_1 = \alpha_2$, and making the further stipulations that

$$M_{L2} = 0$$

$$M_{L1} = M_{TG2}$$

$$J_{p_1} \beta_1 = J_{p_2} \dot{\beta}_2 \qquad \text{(Eq 6. 2. 15)}$$

$$F_1 = F_2$$

which implies that the two controllers are geared together gives

$$\dot{\gamma}_1 = \frac{M_{TG1} - 2 J_p \dot{\beta} p C \gamma_1 C\alpha}{2 F_1} \qquad \text{(Eq 6. 2. 16)}$$

The assumption that $\dot{\alpha}_1 = \dot{\alpha}_2$ is not exact because the α_1 and α_2 gimbal axes cannot be easily geared together, but if the variables are enclosed in an error position control loop the assumption is believed valid. Note that the gearing of controllers 1 and 2 completely eliminates the cross coupling effects of q and $\dot{\alpha}$, and leaves only the primary attitude rate variable, p, of the loop. This is an important advantage of a system using two controllers operating back-to-back.

In arriving at the result of equation 6. 2. 16 is must not be overlooked that an important assumption is that the angles α_1 and α_2 are maintained approximately equal. This assumption gives the two-degree-of-freedom controllers the characteristics of a single-degree-of-freedom controller in that the α axis is approximately rigidly restrained unless all four of the controllers move in the same direction as a unit. In principle this may be accomplished by wiring the torque generators controlling the α axis in parallel and requiring the torque generators to have a high back emf.

Combining controllers 3 and 4 under the conditions that the two are geared together and driven by controller 3 gives for similar assumptions to equation 6. 2. 15 that

$$\dot{\gamma}_3 = \frac{M_{TG3} + 2J_p \dot{\beta} q C\gamma_3 C\alpha}{2F_3} \qquad \text{(Eq 6. 2.17)}$$

Note that in this equation for pitch control that the cross coupling effects of roll and yaw do not appear in the equation.

6. 3 Response of the Controllers to Moments Applied by the Yaw Torque Generators

Following the procedures of the previous section

$$\left.\frac{dH}{dt}\right]_I = \sum \left. M_{gim}\right]_I \qquad \text{(Eq 6. 3. 1)}$$

Then it may be written that

$$\sum \left. M_{gim}\right]_{GIM} = \left.\dot{H}\right]_{GIM} + W_{I,GIM} \times \left. H\right]_{GIM} \qquad \text{(Eq 6. 3. 2)}$$

where $H = H_r + H_g$

Now from Appendix D the angular velocity is

$$W_{I,GIM} = \begin{bmatrix} pC\gamma + qS\gamma + \dot{\alpha} \\ (-pS\gamma + qC\gamma)C\alpha + (r + \dot{\gamma})S\alpha \\ (pS\gamma - qC\gamma)S\alpha + (r + \dot{\gamma})C\alpha \end{bmatrix} \qquad \text{(Eq 6. 3. 3)}$$

Using the approximation of equation 6. 2. 4 one would first guess that $H_{rg}\rfloor_{GIM}$ vanishes; however, such is not the case and we must use the following from Appendix E

$$\dot{H}\rfloor_{GIM} \approx \begin{bmatrix} J_{rg}\ddot{\alpha} \\ 0 \\ 0 \end{bmatrix} \qquad \text{(Eq 6. 3. 4)}$$

This step can be justified by considering the complete equations in Appendix E.

$$\sum M_{gim\,1}\rfloor_{GIM} = \begin{bmatrix} J_{rg}\ddot{\alpha} - (pS\gamma - qC\gamma)S\alpha + (r + \dot{\gamma})C\alpha \quad J_p\dot{\beta} \\ 0 \\ \left[pC\gamma + qS\gamma + \dot{\alpha}\right] J_p\dot{\beta} \end{bmatrix}$$

(Eq 6. 3. 5)

The Y component vanishes only for the assumptions made herein, which means there are no large torques transmitted about the spin axis of the rotor. Here we are interested only in the X component since the Z component will be transferred through the gimbal to the case where a component of it will appear at the γ axis. Note that the cosine α component can be identified in

the corresponding portion of equation 6. 2. 9.

Unfortunately, there does not appear to be a simple method of mechanically connecting the gimbals of the four controllers so that each of these angles are exactly the same. Therefore, each of the gimbals must be handled separately. These may be written as follows

Controller 1 Let $G_1 = J_{rg}\frac{d}{dt} + C$

$$\dot{\alpha}_1 = \frac{M_{TG1} + J_p\dot{\beta}\ (pS\gamma_1 - qC\gamma_1)\,S\alpha_1 + (r + \dot{\gamma}_1)C\alpha_1}{G_1}$$

(Eq 6. 3. 6)

Controller 2

$$\dot{\alpha}_2 = \frac{M_{TG2} + J_p\dot{\beta}\ (pS\gamma_1 + qC\gamma_1)\,S\alpha_2 + (r - \dot{\gamma}_1)\,C\alpha_2}{G_2}$$

(Eq 6. 3. 7)

Controller 3

$$\dot{\alpha}_3 = \frac{M_{TG3} + J_p\dot{\beta}\ (pC\gamma_3 - qS\gamma_3)\,S\alpha_3 + (r + \dot{\gamma}_3)\,C\alpha_3}{G_3}$$

(Eq 6. 3. 8)

Controller 4

$$\dot{\alpha}_4 = \frac{M_{TG4} + J_p\dot{\beta}\ (-pC\gamma_3 - qS\gamma_3)\,S\alpha_4 + (r - \dot{\gamma}_3)\,C\alpha_4}{G_4}$$

(Eq 6. 3. 9)

In the above four equation it is assumed that $\gamma_2 = -\gamma_1$, $\gamma_4 = -\gamma_3$ and that $J_p \dot{\beta}$ is held constant.

If it is assumed momentarily that the α gimbal axes can be mechanically geared together then one may sum the yaw moments of the four controllers. Since the angular momentum of each controller is identical a summation of yaw moments can be accomplished by first finding the average gimbal rate by summing the four gimbal rates. This gives the following equation.

$$\dot{\alpha} = \frac{2 M_{TG} + J_p \dot{\beta} \left[(pS\gamma_1 - qS\gamma_3) S\alpha + 2 r C\alpha \right]}{2G} \qquad \text{(Eq 6. 3. 10)}$$

In this expression the moment applied by the torque generator, M_{TG} , is applied to each of the four gimbal axes. Note that unlike the previous equations for $\dot{\gamma}_1$ and $\dot{\gamma}_3$ the cross-coupling rates p and q do not vanish although they are multiplied by the product of two angles which may remain small. However, the large cosine components of the cross-coupling rates do drop from the equation as do the terms in $\dot{\gamma}_1$ and $\dot{\gamma}_3$.

It is proposed to gear the α gimbals together electrically by finding the average gimbal angle and comparing this with the angle of each gimbal separately in an error feedback loop. The equations for this operation are contained in the next section.

6.4 Torque Generator

The control power requirements for the gyro controllers are insignificant compared to the power required to drive the rotors at operating speed as will be shown, and since the peak torque requirements are small, either a direct current motor or a two phase alternating current motor is applicable. The ideal torque-speed characteristics of both types are the same to a first approximation.

$$J_M \ddot{\theta} + B\dot{\theta} + M_L = K_T V_M \qquad \text{(Eq. 6.4.1)}$$

where
- θ = motor position angle
- J_M = motor moment of inertia
- B = viscous friction constant
- M_L = load torque
- K_T = torque constant
- V_M = input control voltage

The motor is geared to the controller with a gear ratio $\rho > 1$; therefore, the torque applied to the controller gimbal is

$$M_{TG} = \rho M_L \qquad \text{(Eq. 6.4.2)}$$

The angular relation between the motor position angle and gimbal position angle is

$$\theta = \rho\alpha \quad \text{for yaw control}$$
$$\theta = \rho\gamma \quad \text{for roll and pitch control}$$

(Eq. 6.4.3)

This gives for yaw control

$$M_{TG} = K_T \rho V_M - (J_M \rho^2 \ddot{\alpha} + B\rho^2 \dot{\alpha})$$

(Eq. 6.4.4)

and for roll and pitch control

$$M_{TG} = K_T \rho V_M - \left(J_M \rho^2 \ddot{\gamma} + B \rho^2 \dot{\gamma}\right) \qquad \text{(Eq. 6.4.5)}$$

Combining equations 6.2.16 and 6.4.5 gives for roll control

$$\dot{\gamma}_1 = \frac{K_T \rho V_M - 2 J_P \dot{\beta} p C\gamma_1 C\alpha}{F_{1\gamma}} \qquad \text{(Eq. 6.4.6)}$$

where

$$F_{1\gamma} = \left(2 J_{C\,gr} + J_M \rho^2\right)\frac{d}{dt} + \left(2 C + B \rho^2\right)$$

Likewise for pitch control

$$\dot{\gamma}_3 = \frac{K_T \rho V_M + 2 J_P \dot{\beta} q C\gamma_3 C\alpha}{F_{3\gamma}} \qquad \text{(Eq. 6.4.7)}$$

For yaw control the four controllers give separately the following relations.

Let $G_{1\alpha} = \left(J_{rg} + J_M \rho^2\right)\frac{d}{dt} + \left(C + B \rho^2\right)$

$$\dot{\alpha}_1 = \frac{K_T \rho V_M + J_P \dot{\beta} \left[\left(p S\gamma_1 - q C\gamma_1\right) S\alpha_1 + \left(r + \dot{\gamma}_1\right) C\alpha_1\right]}{G_{1\alpha}}$$

(Eq. 6.4.8)

$$\dot{\alpha}_2 = \frac{K_T \rho V_M + J_P \dot{\beta} \left[\left(p S\gamma_1 + q C\gamma_1\right) S\alpha_2 + \left(r - \dot{\gamma}_1\right) C\alpha_2\right]}{G_{2\alpha}}$$

(Eq. 6.4.9)

$$\dot{\alpha}_3 = \frac{K_T \rho V_M + J_P \dot{\beta} \left[\left(p\, C\gamma_3 - q\, S\gamma_3 \right) S\alpha_3 + \left(r + \dot{\gamma}_3 \right) C\alpha_3 \right]}{G_{3\alpha}}$$

(Eq. 6. 4. 10)

$$\dot{\alpha}_4 = \frac{K_T \rho V_M + J_P \dot{\beta} \left[\left(-p\, C\gamma_3 - q\, S\gamma_3 \right) S\alpha_4 + \left(r - \dot{\gamma}_4 \right) C\alpha_4 \right]}{G_{4\alpha}}$$

(Eq. 6. 4. 11)

In the above equations subscripts have been omitted, but each of the equations requires its own characteristic parameters.

Since all controllers are slaved to the position angle of the average controller, the relation for the motor voltage of a typical controller is as follows.

$$V_M = V_{MZ} - K\theta \rho \left(\alpha_1 - \alpha_{ave} \right) \qquad \text{(Eq. 6. 4. 12)}$$

where V_{MZ} = error voltage and $K\theta$ = feedback constant.

Substituting equation 6. 4. 12 into the above four equations gives the following when written in terms of position angles.

Controller 1

$$\alpha_1 = \frac{K_T \rho V_{MZ} + K_\theta \rho \alpha_{ave} + J_P \dot{\beta} \left[\left(p\, S\gamma_1 - q\, C\gamma_1 \right) S\alpha_1 + \left(r + \dot{\gamma}_1 \right) C\alpha_1 \right]}{\frac{d}{dt}\left(G_{1\alpha} \right) + K_\theta \rho}$$

(Eq. 6. 4. 13)

Controller 2

$$\alpha_2 = \frac{K_T \rho V_{MZ} + K_\theta \rho \alpha_{ave} + J_P \dot{\beta} \left[\left(p\, S\gamma_1 + q\, C\gamma_1 \right) S\alpha_2 + \left(r - \dot{\gamma}_1 \right) C\alpha_2 \right]}{\frac{d}{dt}\left(G_{2\alpha} \right) + K_\theta \rho}$$

(Eq. 6. 4. 14)

Controller 3

$$\alpha_3 = \frac{K_T \rho V_{MZ} + K_\theta \rho \alpha_{ave} + J_P \dot{\beta}\left[\left(p C\gamma_3 - q S\gamma_3\right) S\alpha_3 + \left(r + \dot{\gamma}_3\right) C\alpha_3\right]}{\frac{d}{dt}\left(G_{3\alpha}\right) + K_\theta \rho}$$

(Eq. 6. 4. 15)

Controller 4

$$\alpha_4 = \frac{K_T \rho V_{MZ} + K_\theta \rho \alpha_{ave} + J_P \dot{\beta}\left[\left(-p C\gamma_3 - q S\gamma_3\right) S\alpha_4 + \left(r - \dot{\gamma}_3\right) C\alpha_4\right]}{\frac{d}{dt}\left(G_{4\alpha}\right) + K_\theta \rho}$$

(Eq. 6. 4. 16)

If we assume that the gimbal angles are held approximately equal such that $C\alpha_1 = C\alpha_2 = C\alpha_3 = C\alpha_4$, then the operation of finding the control moments contributed by the four controllers can be accomplished by first finding the average value of the gimbal rates by combining the last four equations. This gives for the average gimbal rate the following

$$\dot{\alpha} = \frac{2K_T \rho V_{MZ} + J_P \dot{\beta}\left[\left(p S\gamma_1 - q S\gamma_3\right) S\alpha + 2 r C\alpha\right]}{2 G_\alpha}$$

(Eq. 6. 4. 17)

Where $G_2 = \left(J_{rg} + J_M \rho^2\right) \frac{d}{dt} + \left(C + B \rho^2\right)$

6.5 Attitude Sensors

The primary source of guidance, navigation, and attitude information for a spacecraft is from external sightings. As was shown in Chapter 4 the ambient fields around a spacecraft are too feable to drive a practical sensor, and this implies that some type of star tracker must be provided if the spacecraft is to be self sufficient. A control system can control the attitude of a spacecraft only to the precision provided by the attitude sensor, and the design of a highly linear sensor is a problem within itself. As done by other investigations of hardware for applying torques to a spacecraft, this thesis assumes a linear sensor for each of the three attitude reference axes of the spacecraft. Accordingly the following equation is assumed to represent the sensor.

$$\begin{bmatrix} K_{sx} K_{ex} & 0 & 0 \\ 0 & K_{sy} K_{ey} & 0 \\ 0 & 0 & K_{sz} K_{ez} \end{bmatrix} \begin{bmatrix} \phi_r - \phi \\ \theta_r - \theta \\ \psi_r - \psi \end{bmatrix} = \begin{bmatrix} V_{cx} \\ V_{cy} \\ V_{cz} \end{bmatrix} \qquad \text{(Eq 6. 5. 1)}$$

K_s = sensitivity of sensor

K_e = sensitivity of error amplifier

ϕ_r = reference angle

ϕ = vehicle attitude angle

V_c = voltage signal to tandem compensation

Any practical sensor has linear characteristics over only a small range, and the effects of saturation will be considered in a later section concerning the position control mode.

6.6 Tandem Compensation

To accommodate the provision for shaping the error signal before it is fed to the control system torque generators, a transfer function which is presently undefined is inserted into each control loop. The following equation applies.

$$\begin{bmatrix} C_X & 0 & 0 \\ 0 & C_Y & 0 \\ 0 & 0 & C_Z \end{bmatrix} \begin{bmatrix} V_{CX} \\ V_{CZ} \\ V_{CZ} \end{bmatrix} = \begin{bmatrix} V_{MX} \\ V_{MY} \\ V_{MZ} \end{bmatrix}$$

(Eq. 6.6.1)

V_c = voltage signal to tandem compensation

C = transfer function of compensation

V_M = voltage signal to torque generators

It is not anticipated that any tandem compensation will be required in the examples of this thesis. In an actual problem certain regions of the s-plane may be denied to the control system designer because of unstable modes of the airframe, etc., and for this reason space in the equations has been provided for tandem compensation.

6.7 Control System Torques Applied to the Spacecraft

The control moments applied to the vehicle by the four gyro controller can be determined by substituting equations 6.4.6, 6.4.7, and 6.4.17 into equation G.2.11 which gives the following.

$$\sum M_{(12-34-1234)}\Big]_A = 2 J_p \dot{\beta} \begin{bmatrix} -\dfrac{K_{tx} \rho_x C\alpha C\gamma_1}{F_{1\gamma}} & 0 & +\dfrac{K_{tz} \rho_z S\alpha S\gamma_1}{G_\alpha} \\ 0 & +\left\{\dfrac{K_{ty} \rho_y C\alpha C\gamma_3}{F_{3\gamma}}\right\} & -\dfrac{K_{tz} \rho_z S\alpha S\gamma_3}{G_\alpha} \\ 0 & 0 & \dfrac{2 K_{tz} \rho_z C\alpha}{G_\alpha} \end{bmatrix} \begin{bmatrix} V_{mx} \\ V_{my} \\ V_{mz} \end{bmatrix} +$$

$$+ \left(2 J_p \dot{\beta}\right)^2 \begin{bmatrix} \left\{\dfrac{(C\gamma_1 C\alpha)^2}{F_{1\gamma}} + \dfrac{(S\alpha S\gamma_1)^2}{4 G_\alpha}\right\} & \left\{\dfrac{S\alpha}{J_p \dot{\beta}} - \dfrac{(S\alpha)^2 S\gamma_1 S\gamma_3}{4 G_\alpha}\right\} & \left\{-\dfrac{C\alpha S\gamma_3}{2 J_p \dot{\beta}} + \dfrac{S\alpha S\gamma_1 C\alpha}{2 G_\alpha}\right\} \\ \left\{-\dfrac{S\alpha}{J_p \dot{\beta}} - \dfrac{(S\alpha)^2 S\gamma_1 S\gamma_3}{4 G_\alpha}\right\} & \left\{\dfrac{(C\gamma_3 C\alpha)^2}{F_{3\gamma}} + \dfrac{(S\alpha S\gamma_3)^2}{4 G_\alpha}\right\} & \left\{-\dfrac{C\alpha S\gamma_1}{2 J_p \dot{\beta}} - \dfrac{S\alpha C\alpha S\gamma_3}{2 G_\alpha}\right\} \\ \left\{\dfrac{C\alpha S\gamma_3}{2 J_p \dot{\beta}} + \dfrac{S\alpha C\alpha S\gamma_1}{2 G_\alpha}\right\} & \left\{\dfrac{C\alpha S\gamma_1}{2 J_p \dot{\beta}} - \dfrac{S\alpha C\alpha S\gamma_3}{2 G_\alpha}\right\} & \left\{\dfrac{(C\alpha)^2}{G_\alpha}\right\} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq 6.7.1)

6.8 Position Control Equations

The closed loop equation for position control may be written by first substituting equations 6.5.1 and 6.6.1 into equation 6.7.1 to obtain the following.

Let $J_p \dot{\beta} = H$

$$K_x = K_{sx}K_{ex}K_{tx}\rho_x C_x \ , \quad K_y = -K_{sy}K_{ey}K_{ty}\rho_y C_y \ , \quad K_z = -K_{sz}K_{ez}K_{tz}\rho_z C_z$$

$$\sum M_{(12-34-1234)}\Big]_A = 2H \begin{bmatrix} -\dfrac{C\alpha C\gamma_1 K_x}{F_{1\gamma}} & 0 & -\dfrac{S\alpha S\gamma_1 K_z}{G_\alpha} \\ 0 & -\dfrac{C\alpha C\gamma_3 K_y}{F_{3\gamma}} & +\dfrac{S\alpha S\gamma_3 K_z}{G_\alpha} \\ 0 & 0 & -\dfrac{2C\alpha K_z}{G_\alpha} \end{bmatrix} \begin{bmatrix} \phi_r - \phi \\ \theta_r - \theta \\ \psi_r - \psi \end{bmatrix}$$

$$+4H^2 \begin{bmatrix} \left\{\dfrac{(C\gamma_1 C\alpha)^2}{F_{1\gamma}} + \dfrac{(S\alpha S\gamma_1)^2}{4G_\alpha}\right\} & \left\{\dfrac{S\alpha}{H} - \dfrac{(S\alpha)^2 S\gamma_1 S\gamma_3}{4G_\alpha}\right\} & \left\{-\dfrac{C\alpha S\gamma_3}{2H} + \dfrac{S\alpha S\gamma_1 C\alpha}{2G_\alpha}\right\} \\ \left\{-\dfrac{S\alpha}{H} - \dfrac{(S\alpha)^2 S\gamma_1 S\gamma_3}{4G_\alpha}\right\} & \left\{\dfrac{(C\gamma_3 C\alpha)^2}{F_{3\gamma}} + \dfrac{(S\alpha S\gamma_3)^2}{4G_\alpha}\right\} & \left\{-\dfrac{C\alpha S\gamma_1}{2H} - \dfrac{S\alpha C\alpha S\gamma_3}{2G_\alpha}\right\} \\ \left\{\dfrac{C\alpha S\gamma_3}{2H} + \dfrac{S\alpha C\alpha S\gamma_1}{2G_\alpha}\right\} & \left\{\dfrac{C\alpha S\gamma_1}{2H} - \dfrac{S\alpha C\alpha S\gamma_3}{2G_\alpha}\right\} & \left\{\dfrac{(C\alpha)^2}{G_\alpha}\right\} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \qquad \text{(Eq 6.8.1)}$$

For convenience we write equation 6.8.1 in shorthand notation as follows.

$$\sum M_{(12-34-1234)}\Big]_A = 2H\left[A\right]\begin{bmatrix}\phi_r - \phi \\ \theta_r - \theta \\ \psi_r - \psi\end{bmatrix} + 4H^2\left[B\right]\begin{bmatrix}p \\ q \\ r\end{bmatrix}$$

(Eq 6.8.2)

where $\left[A\right]$ and $\left[B\right]$ are the indicated matrices and (a_{ij}) and (b_{ij}) are typical elements of these matrices.

To obtain the equation for the complete closed loop system we substitute equations 6.8.2 and 2.4.5 into equation 2.3.3 which gives the following.

$$\begin{bmatrix} \left\{ I_x \frac{d^2}{dt^2} + 4H^2 b_{11}\frac{d}{dt} - 2Ha_{11} \right\} & \left\{ 4H^2 b_{12}\frac{d}{dt} \right\} & \left\{ 4H^2 b_{13}\frac{d}{dt} - 2Ha_{13} \right\} \\ \left\{ 4H^2 b_{21}\frac{d}{dt} \right\} & \left\{ I_y \frac{d^2}{dt^2} + 4H^2 b_{22}\frac{d}{dt} - 2Ha_{22} \right\} & \left\{ 4H^2 b_{23}\frac{d}{dt} - 2Ha_{23} \right\} \\ \left\{ 4H^2 b_{31}\frac{d}{dt} \right\} & \left\{ 4H^2 b_{32}\frac{d}{dt} \right\} & \left\{ I_z \frac{d^2}{dt^2} + 4H^2 b_{33}\frac{d}{dt} - 2Ha_{33} \right\} \end{bmatrix} \begin{bmatrix}\phi \\ \theta \\ \psi\end{bmatrix}$$

$$+ \begin{bmatrix}(I_z - I_y)qr \\ (I_x - I_z)pr \\ (I_y - I_x)qp\end{bmatrix} = \sum M_{ext}\Big]_A - \sum M_{VM}\Big]_A - 2H\left[A\right]\begin{bmatrix}\phi_r \\ \theta_r \\ \psi_r\end{bmatrix}$$

(Eq 6.8.3)

Equation 6.8.2 represents the complete loop of the attitude control system, and as can be seen there is non-linear interaxis coupling between the three loops as well as non-linear terms arising from the Euler inertia-cross coupling. If it is assumed that compensation is applied to the loops as set forth in section 3.6, and also compensation for the inertia-cross coupling terms, the following expression is found for the roll attitude angle.

$$I_x \frac{d^2\phi}{dt^2} + 4H^2 b_{11} \frac{d\phi}{dt} - 2Ha_{11}\phi = M_x - 2Ha_{11}\phi_r \qquad \text{(Eq 6.8.4)}$$

Similar expressions can be written for θ and ψ.

If the angular motions of the α axes of the controllers are restricted to small angles then the following equations apply.

$$I_x F_{1\gamma} \frac{d^2\phi}{dt^2} + (2HC\gamma_1)^2 \frac{d\phi}{dt} + 2HC\gamma_1 K_x \phi = F_1 \gamma M_x + 2HC\gamma_1 K_x \phi_r$$

(Eq 6.8.5)

$$I_y F_{3\gamma} \frac{d^2\theta}{dt^2} + (2HC\gamma_3)^2 \frac{d\theta}{dt} + 2HC\gamma_3 K_y \theta = F_3 \gamma M_y + 2HC\gamma_3 K_y \theta_r$$

(Eq 6.8.6)

$$I_z G_\alpha \frac{d^2\psi}{dt^2} + (2HC\alpha)^2 \frac{d\psi}{dt} + 4HC\alpha K_z \psi = G\alpha M_z + 4HC\alpha K_z \psi_r$$

(Eq 6.8.7)

where $F_{1\gamma} = (2J_{cgr} + J_M \rho^2)\frac{d}{dt} + (2C + B\rho^2)$

$$F_{3\gamma} = (2J_{cgr} + J_M \rho^2)\frac{d}{dt} + (2C + B\rho^2)$$

$$G_\alpha = (J_{gr} + J_M \rho^2)\frac{d}{dt} + (C + B\rho^2)$$

the constants for the different axes may not be equal.

Block diagram representation for the above equations are contained in Figure 6.8.1 and Figure 6.8.2. Figure 6.8.1 contains the roll and pitch channels whereas the yaw channel is shown in Figure 6.8.2 in two parts. Part (a) shows the four individual controllers driven by the same error voltage. To keep these controllers to the same angle their output angles are averaged and then compared with the actual output gimbal angle. The error is then fed back to the gimbal torque motor. Part (b) of Figure 6.8.2 is the result of ideal controllers and represents equation 6.8.7.

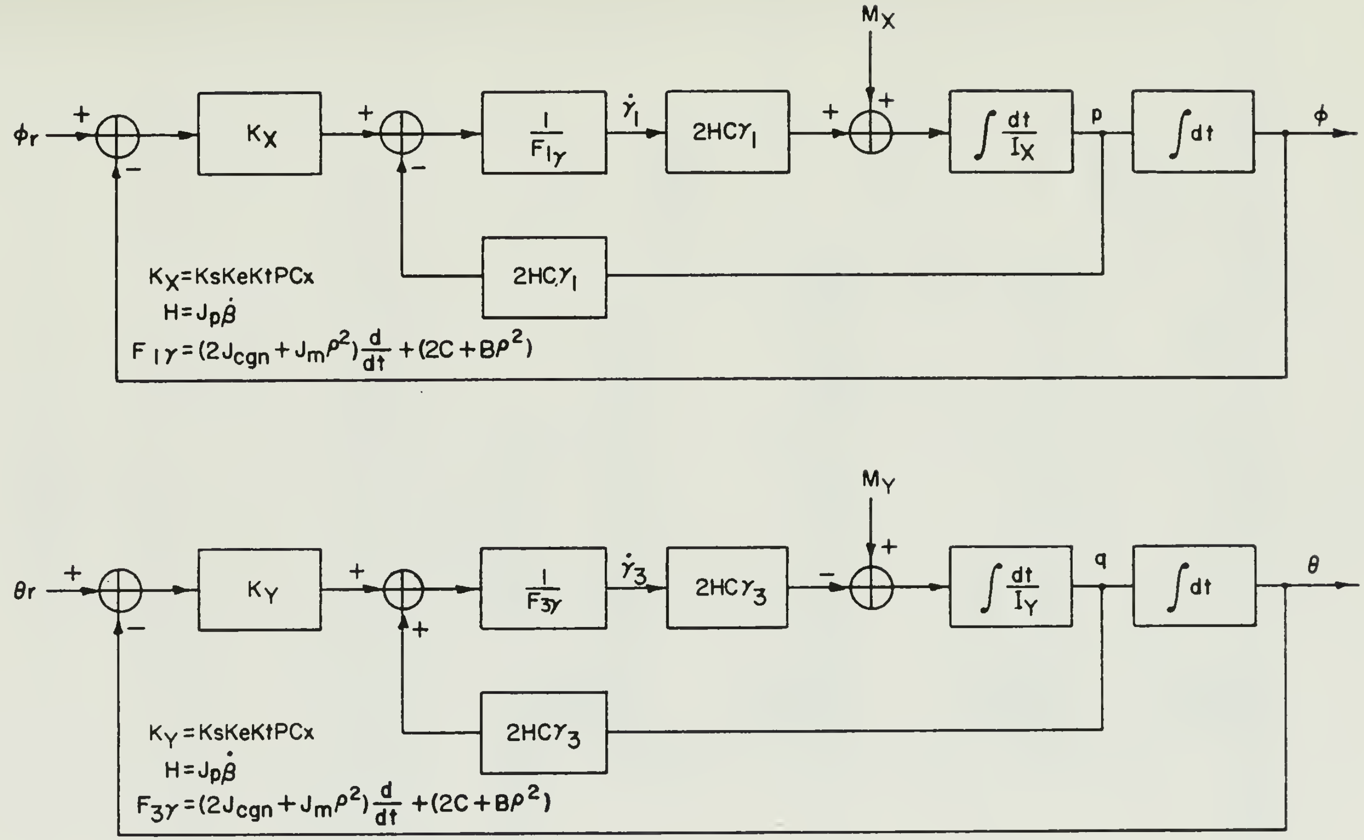
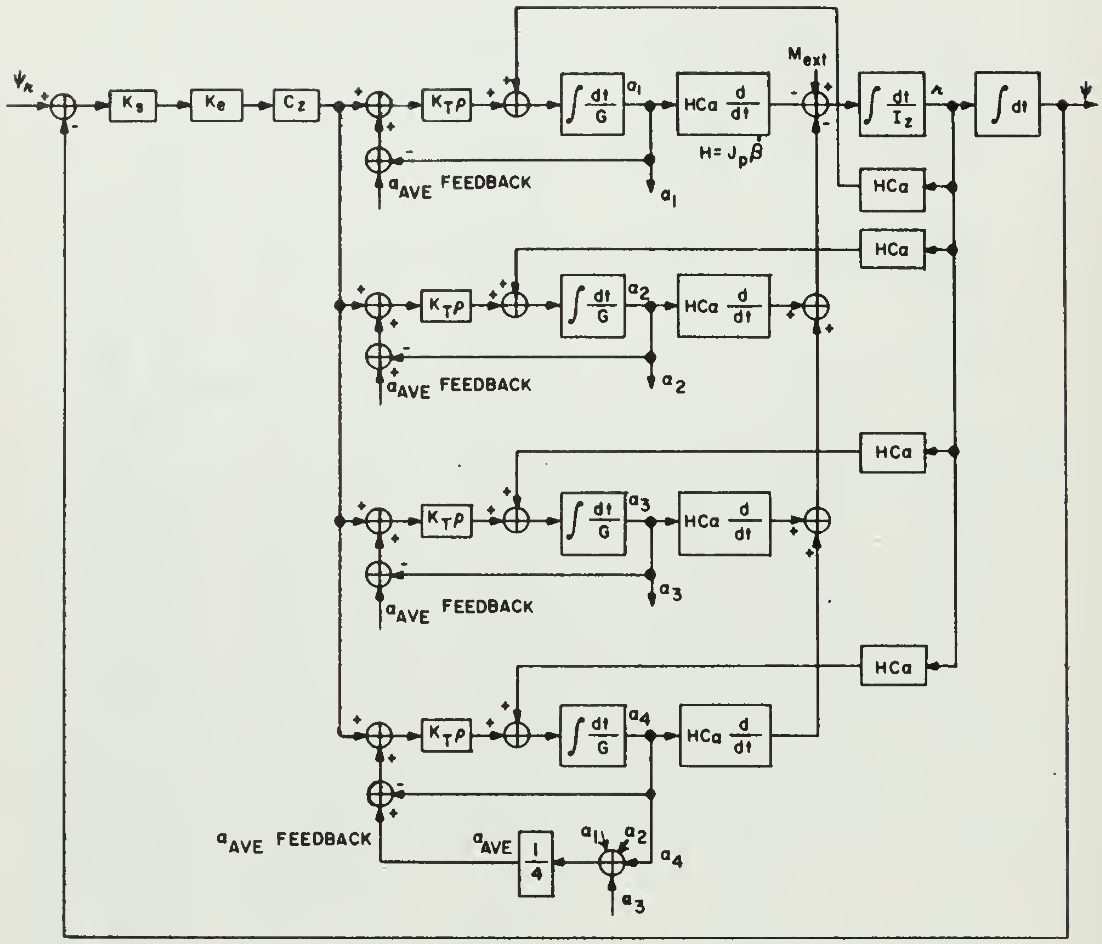
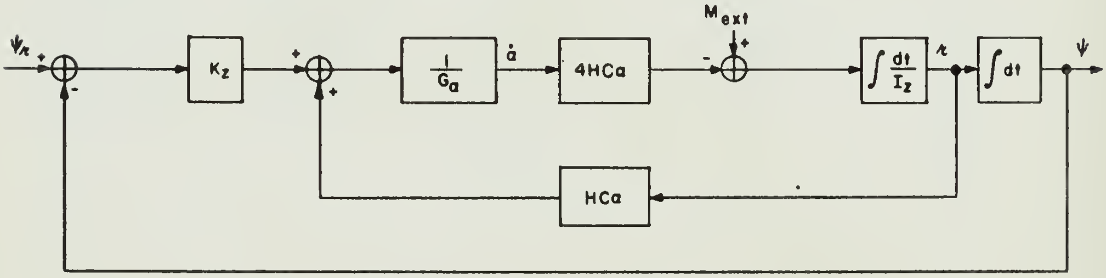


Figure 6.8.1 Block Diagram Representation of the Position Control Roll Equation and the Pitch Equation for Uncoupled Motion. See Equations 6.8.5 and 6.8.6.



PART (a) INDIVIDUAL CONTROLLERS AND AVERAGING CIRCUIT SHOWN



PART (b) SIMPLIFIED REPRESENTATION OF EQUATION 6.8.7

Figure 6.8.2 Block Diagram Representation of the Position Control Yaw Equation for Uncoupled Motion.

6.9 Rate Control Equations

The purpose of the rate control is to provide the astronaut with a means of slewing the vehicle at a particular attitude rate. Since the position of the gimbal angles are directly related to the amount of control system angular momentum transferred to the vehicle, the gimbal angles represent attitude rates. Therefore the rate control is essentially an error feedback control loop to control the gimbal angle, and the block diagram of the roll channel is shown in Figure 6.9.1. During this operation the error signals from the vehicle attitude sensors are open circuited.

During the rate control maneuver the voltages to the torque motors are given by the following equation.

$$\begin{bmatrix} V_{mx} \\ V_{my} \\ V_{mz} \end{bmatrix} = \begin{bmatrix} K_{\gamma 1} & 0 & 0 \\ 0 & K_{\gamma 3} & 0 \\ 0 & 0 & K_{\alpha} \end{bmatrix} \begin{bmatrix} \gamma_{1r} - \gamma_1 \\ \gamma_{3r} - \gamma_3 \\ \alpha_r - \alpha \end{bmatrix} \qquad \text{(Eq 6.9.1)}$$

The voltages given by equation 6.9.1 may be substituted into equations 6.4.6, 6.4.7, and 6.4.17 to find an expression for the gimbal angles as a function of the rate variables and the rate inputs. For shortness we write

$$K_1 = K_{\gamma 1} K_{tx} \rho_x$$

$$K_2 = K_{\gamma 3} K_{ty} \rho_y$$

$$K_3 = K_{\alpha} K_{t\alpha} \rho_{\alpha}$$

which gives the following matrix equation.

$$
\begin{bmatrix} F_{1\gamma}\frac{d}{dt} + K_1 & 0 & 0 \\ 0 & F_{3\gamma}\frac{d}{dt} + K_2 & 0 \\ 0 & 0 & G\frac{d}{dt} + K_3 \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_3 \\ \alpha \end{bmatrix} =
$$

$$
\begin{bmatrix} K_1\gamma_{1r} \\ K_2\gamma_{3r} \\ K_3\alpha_r \end{bmatrix} + 2H \begin{bmatrix} -C\gamma_1 C\alpha & 0 & 0 \\ 0 & +C\gamma_3 C\alpha & 0 \\ +\frac{S\gamma_1 S\alpha}{4} & -\frac{S\gamma_3 S\alpha}{4} & +\frac{C\alpha}{4} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \qquad \text{(Eq 6.9.2)}
$$

The above matrix equation must be solved for the gimbal angle rates so that these rates can be applied to the control system represented by equation G.2.11. Because of the non-linearities caused by the sine and cosine terms of equation 6.9.2, the Laplace techniques are not simple to apply unless the angles can be considered constant. Therefore, in the equations that follow the notation employs the letter s so that a closed form solution can be written, but this symbol s refers to the Laplacian operator only in the special case where sine and cosine values of the gimbal angles can be considered constant. Otherwise, for cases where the gimbal angles make large changes, the symbol s must be interpreted as a differentiation with respect to time.

Solving the matrix equation 6.9.2 for the gimbal angle rates and substituting in equation G.2.11 gives the following expression for the moments generated by the spacecraft control system.

$$\sum M_{\substack{\text{Rate}\\\text{Control}}}\Big]_A = 2H \begin{bmatrix} -\dfrac{C\alpha C\gamma_1 SK_1}{SF_{1\gamma} + K_1}\,\gamma_{1r} \\ +\dfrac{C\alpha C\gamma_3 SK_2}{SF_{3\gamma} + K_2}\,\gamma_{3r} \\ +\dfrac{2\,C\alpha\, SK_3}{SG_\alpha + K_3}\,\alpha_r \end{bmatrix} +$$

$$(2H)^2 \begin{bmatrix} \left\{\dfrac{(C\alpha C\gamma_1)^2 S}{SF_{1\gamma} + K_1} + \dfrac{1}{4}\dfrac{(S\alpha S\gamma_1)^2 S}{SG_\alpha + K_3}\right\} & \left\{-\dfrac{1}{4}\dfrac{S^2\alpha S\gamma_1 S\gamma_3}{SG_\alpha + K_3} + \dfrac{S\alpha}{H}\right\} & \left\{+\dfrac{1}{2}\dfrac{S\alpha C\alpha S\gamma_1 S}{SG_\alpha + K_3} - \dfrac{C\alpha S\gamma_3}{2H}\right\} \\ \left\{-\dfrac{1}{4}\dfrac{S^2\alpha S\gamma_1 S\gamma_3 S}{SG_\alpha + K_3} - \dfrac{S\alpha}{H}\right\} & \left\{\dfrac{(C\alpha C\gamma_3)^2 S}{SF_{3\gamma} + K_2} + \dfrac{1}{4}\dfrac{(S\alpha S\gamma_3)^2 S}{SG_\alpha + K_3}\right\} & \left\{-\dfrac{1}{2}\dfrac{S\alpha C\alpha S\gamma_3 S}{SG_\alpha + K_3} - \dfrac{C\alpha S\gamma_1}{2H}\right\} \\ \left\{+\dfrac{1}{2}\dfrac{S\alpha C\alpha S\gamma_1 S}{SG_\alpha + K_3} + \dfrac{C\alpha S\gamma_3}{2H}\right\} & \left\{-\dfrac{1}{2}\dfrac{S\alpha C\alpha S\gamma_3 S}{SG_\alpha + K_3} + \dfrac{C\alpha S\gamma_1}{2H}\right\} & \left\{\dfrac{(C\alpha)^2 S}{SG_\alpha + K_3}\right\} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

where $H = J_p \dot{\beta}$ (Eq 6.9.3)

As in the previous solution, rewrite equation 6.9.3 as follows.

$$\sum M_{\substack{\text{Rate}\\\text{Control}}}\Big]_A = 2H \begin{bmatrix} \left(\dfrac{-C\alpha C\gamma_1 SK_1}{SF_{1\gamma} + K_1}\right)\gamma_{1r} \\ \left(\dfrac{C\alpha C\gamma_3 SK_2}{SF_{3\gamma} + K_2}\right)\gamma_{3r} \\ \left(\dfrac{2C\alpha SK_3}{SG_\alpha + K_3}\right)\alpha_r \end{bmatrix} + 4H^2 [E] \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq 6.9.4)

Substituting equation 6.9.3 and 2.4.5 into equation 2.3.3 gives the following equation for the response to rate control.

$$\begin{bmatrix} \left(I_x S + 4H^2 e_{11}\right) & \left(4H^2 e_{12}\right) & \left(4H^2 e_{13}\right) \\ \left(4H^2 e_{21}\right) & \left(I_y S + 4H^2 e_{22}\right) & \left(4H^2 e_{23}\right) \\ \left(4H^2 e_{31}\right) & \left(4H^2 e_{32}\right) & \left(I_z S + 4H^2 e_{33}\right) \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \begin{bmatrix} \left(I_z - I_y\right) qr \\ \left(I_x - I_z\right) pr \\ \left(I_y - I_x\right) qp \end{bmatrix} =$$

$$\sum M_{ext}\Big]_A - \sum M_{VM}\Big]_A - 2H \begin{bmatrix} \dfrac{-C\alpha C\gamma_1 K_1 S}{SF_{1\gamma} + K_1} \gamma_{1r} \\ \dfrac{C\alpha C\gamma_3 K_2 S}{SF_{3\gamma} + K_2} \gamma_{3r} \\ \dfrac{2C\alpha K_3 S}{SG_\alpha + K_3} \alpha_r \end{bmatrix}$$

(Eq 6.9.5)

The uncoupled set of equations for roll, pitch and yaw is given by the following.

$$p = \frac{M_x (SF_{1\gamma} + K_1) + \gamma_{1r} (2\,HC\gamma_1 K_1 S)}{S\left(I_x F_{1\gamma} S + I_x K_1 + (2\,HC\gamma_1)^2\right)} \qquad \text{(Eq 6.9.6)}$$

$$q = \frac{M_y (SF_{3\gamma} + K_2) - \gamma_{3r} (2\,HC\gamma_3 K_2 S)}{S\left(I_y F_{3\gamma} S + I_y K_2 + (2\,HC\gamma_3)^2\right)} \qquad \text{(Eq 6.9.7)}$$

$$r = \frac{M_z (SG_\alpha + K_3) - \alpha (4\,HC\alpha K_3 S)}{S\left(I_z G_\alpha S + I_z K_3 + (2\,HC\gamma_3)^2\right)} \qquad \text{(Eq 6.9.8)}$$

The following equations are valid for the rate input commands which drive the gimbal angles to large values in those cases where there is negligible inter-axial coupling.

For Roll

$$\left.\begin{aligned} F_{1\gamma}\dot{\gamma}_1 + K_1\gamma_1 &= K_1\gamma_{1r} - 2\,HC\gamma_1 p \\ M_x &= -2\,HC\gamma_1\dot{\gamma}_1 + I_x\dot{p} \end{aligned}\right\} \qquad \text{(Eq 6.9.9)}$$

For Pitch

$$\left.\begin{aligned} F_{3\gamma}\dot{\gamma}_3 + K_2\gamma_3 &= K_2\gamma_{3r} + 2\,HC\gamma_3 q \\ M_y &= 2\,HC\gamma_3\dot{\gamma}_3 + I_y\dot{q} \end{aligned}\right\} \qquad \text{(Eq 6.9.10)}$$

For Yaw

$$\left.\begin{aligned} G\dot{\alpha} + K_3\alpha &= K_3\alpha_r + HC\alpha_r \\ M_z &= 4\,HC\alpha\,\alpha + I_z\dot{r} \end{aligned}\right\} \qquad \text{(Eq 6.9.11)}$$

For the completely general case it is necessary to solve equations 6.9.2, G.2.11, 2.4.5, and 2.3.3 by machine computation.

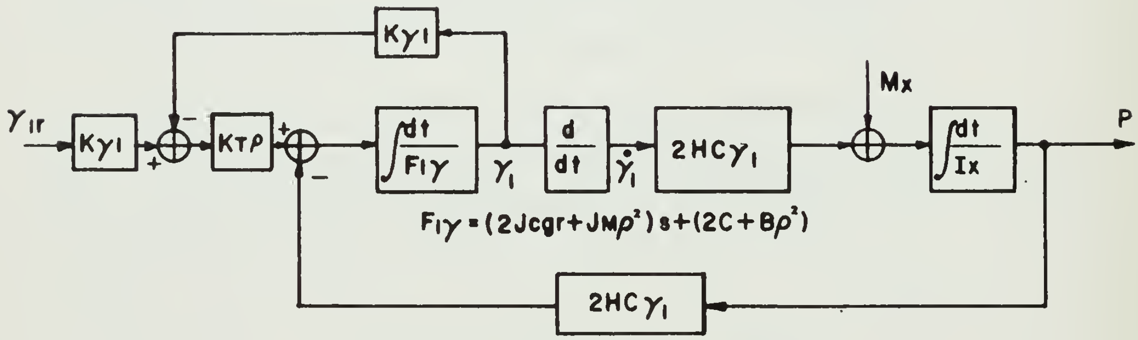


Figure 6.9.1 Block Diagram Representation for Roll Channel of the Uncoupled Rate Control Equation.

6.10 Summary

This chapter has served to derive the closed loop equations of motion of the spacecraft. Examination of the closed loop position control equation 6. 8. 3 and the closed loop rate control equation 6. 9. 5 confirms the fact that the equations are coupled from control interaction as well as the Euler rate-product terms. Therefore, a completely general solution requires assistance from a computer. One of the reasons for choosing the four controller system is that many of the large cross-coupling terms in the primary control matrix vanish, and since the four controller system is a zero momentum type system the cross-coupling terms of the gyroscopic coupling matrix are small. Cross-coupling is a problem in any type of flying machine if we expect to actuate the control of two axes to their full travel simultaneously. Generally speaking vehicles are not operated in this manner, and if crossed controls are required one of the axes will predominate.

In the following chapter the equations of motion are solved to determine the response of the vehicle. This is accomplished by assuming no interaxial coupling, and solving the equations analytically. Making this assumption, many of the equations can be solved in closed form whereas others require graphical methods. It is believed that this approach gives a better understanding and presentation of the system than a pure computer solution which gives graphical response of the vehicle to input disturbances.

CHAPTER 7

DYNAMICS OF THE SPACECRAFT

7.1 Introduction

This chapter evaluates the response of the spacecraft using the equations developed in the preceding pages. This is accomplished by assuming that the interaxial coupling among the three control axes can be neglected. This assumption is valid provided the control system is not appreciably saturated, and that the astronauts do not attempt to simultaneously actuate the controls to yield large rates about two axes. In any event, a necessary condition is that the attitude control system must perform satisfactorily about each of the individual control axes so that initial analysis of the equations of motion on an uncoupled basis is considered reasonable. Also, several parameters of the system must be numerically chosen, and their selection is greatly simplified using uncoupled equations. These parameters include the damping coefficient of the controllers, the gain of the rate control loop, and the gain of the position control loop. It is visualized that the spacecraft will be operated in one of three modes.

1. Zero Input Mode
2. Rate Control Mode
3. Position Control Mode

In addition to these three modes, a mode called the Adaptive Mode will be discussed.

The zero input mode is that which results when no signal is provided to the torque motors of the controllers. The control system then operates as a rate stabilizer, and the gyro controllers provide inherent sensing. The stability of the zero input mode will serve as a criterion for the selection of the damping coefficient of the controllers. The damping coefficient is then held constant

for the other two modes. The purpose of the rate control is to command the spacecraft to perform a particular attitude rate. The stability of the rate control mode will be used to ascertain the gain of the rate control loop, and the stability of the position control mode will determine the gain of this loop. The adaptive mode is the vehicle response to a controller failure.

The damping chosen for the controllers differs appreciably from that used in systems which overdamp the gyro to the point that it can be approximated as a first order system. In the present design the controllers act as a true second order system with two poles having both real and imaginary parts. This of course complicates the solution of the equations, some of which cannot be solved in closed form. Therefore, in many of the equations which follow it is necessary to approximate the second order system by a first order system to arrive at useful results.

As was done in Chapter 5, the analysis will be accomplished using the equations for roll. The equations for pitch control are almost precisely the same as those for roll, and generally the same parameters are required except that the numbers will differ because of different moments of inertia of the spacecraft about the roll and pitch axes. The yaw equations differ slightly from those of roll, and a summary of the differences will be made.

7.2 Zero Input Mode

The uncoupled equations for the zero input motion are given by equations 6.8.5, 6.8.6, and 6.8.7 where the position input terms have been dropped, and these equations are written as follows.

$$p = \frac{F_{1\gamma} \; M_x}{I_x F_{1\gamma} s + 4H^2 \, (C\gamma_1)^2} \qquad \text{(Eq. 7.2.1)}$$

$$q = \frac{F_{3\gamma} \; M_y}{I_y F_{3\gamma} s + 4H^2 \, (C\gamma_3)^2} \qquad \text{(Eq. 7.2.2)}$$

$$r = \frac{G_\alpha \; M_z}{I_z G_\alpha s + 4H^2 \, (C\alpha)^2} \qquad \text{(Eq. 7.2.3)}$$

The stability of the spacecraft for the complete range of gimbal angles can be investigated by assuming small perturbances wherein the gimbal angle can be taken as a constant. Consider the equation for roll motion and let $F_{1\gamma}$ which is actually equal to

$F_{1\gamma} = \left(2J_{cgr} + J_M \rho^2\right) s + \left(2C + B\rho^2\right)$ be written, for shortness, simply as $F_{1\gamma} = Js + k$. This gives

$$p = \frac{M_x \quad (s + k/J)}{I_x \quad s^2 + \frac{ks}{J} + \frac{4H^2 \, (C\gamma_1)^2}{I_x J}} \qquad \text{(Eq. 7.2.4)}$$

The motion follows that of a second order system with a damping ratio

$$\zeta = \frac{k}{4 H C\gamma_1} \sqrt{\frac{I_x}{J}} \qquad \text{(Eq. 7.2.5)}$$

and a natural frequency.

$$\omega_n = \frac{2 H C\gamma_1}{\sqrt{I_x J}} \qquad \text{(Eq. 7.2.5)}$$

The effect of the disturbance is least when the gimbal angle is small for which the following are defined

$$\zeta_o = \frac{k}{4 H} \sqrt{\frac{I_x}{J}} \qquad \text{(Eq. 7.2.6)}$$

$$\omega_o = \frac{2 H}{\sqrt{I_x J}} \qquad \text{(Eq. 7.2.7)}$$

The solution for the response of the system to an impulsive moment, M_o, for the lightly damped controller is given by the following equation.

$$p = \frac{M_o}{I_x} \frac{C\gamma_1}{\sqrt{C\gamma_1^2 - \zeta_o^2}} e^{-\zeta_o \omega_o t} \cdot \sin\left\{ \omega_o \sqrt{C\gamma_1^2 - \zeta_o^2}\, t + \tan^{-1} \frac{\sqrt{C\gamma_1^2 - \zeta_o^2}}{\zeta_o} \right\}$$

(Eq. 7.2.8)

Examination of this equation shows that the roll rate is always stable for any gimbal angle less than 90 degrees because the exponential damping factor is not a function of the gimbal angle. Critical damping occurs when $C\gamma_1 = \zeta_o$ and the solution for this motion is given by the following equation.

$$p = \frac{M_o}{I_x}\left[\zeta_o \omega_o t + 1\right] e^{-\zeta_o \omega_o t}$$

(Eq. 7. 2. 8)

The highly damped motion can be approximated by assuming that the gimbal moment of inertia is negligibly small giving the following equation for roll rate.

$$p = \frac{M_x}{I_x} \frac{1}{\left\{S + \frac{4 H^2}{I_x k}(C\gamma_1)^2\right\}}$$

(Eq. 7. 2. 9)

The solution to this equation for an impulsive moment disturbance is given by

$$p = \frac{M_o}{I_x} e^{-\frac{4 H^2 C\gamma_1^2 t}{I_x k}}$$

(Eq. 7. 2. 10)

In terms of previously defined parameters this motion can be written as follows.

$$p = \frac{M_o}{I_x} e^{\frac{-\omega_n}{2\zeta} t}$$

(Eq. 7. 2. 11)

The analysis of the response due to a step moment disturbance is much more difficult to analyze because the gimbal angle cannot be assumed constant. As a beginning, however, let us assume that the moment is small enough that first order dynamics can be used in which case equation 7. 2. 9 is considered. Further, let the gimbal angle remain small so that the cosine of the gimbal angle can be approximated as a value of one. A step of M_1 at time zero gives the following.

$$p = \frac{M_1 k}{4 H^2} \left(1 - e^{\frac{-\omega_o}{2 \zeta_o} t} \right) \qquad \text{(Eq. 7.2.12)}$$

The final value is seen to be independent of the moment of inertia of the vehicle. The roll rate of a vehicle without an attitude control system in response to a step disturbance moment is given by

$$p = \frac{M_1}{I_x} t \qquad \text{(Eq. 7.2.13)}$$

The roll rate of a spacecraft without a control system compared to that with gyro controllers both of which experience a step moment disturbance is given by dividing equation 7.2.13 by 7.2.12 as follows.

$$\frac{p_{\text{without control system}}}{p_{\text{with gyro controllers}}} = \frac{\omega_o}{2\zeta_o} t \left(\frac{1}{1 - e^{\frac{-\omega_o}{2\zeta_o} t}} \right)$$

(Eq. 7.2.14)

The factor $\frac{\omega_o}{2\zeta_o}$ is the reciprocal of the time constant for the gyro controller and the factor is large for fast controllers, so that the ratio is plotted initially as a steep curve in Figure 7.2.1. The effect of saturation of the gyro controllers is sketched in to show that the ratio approaches unity as time increases.

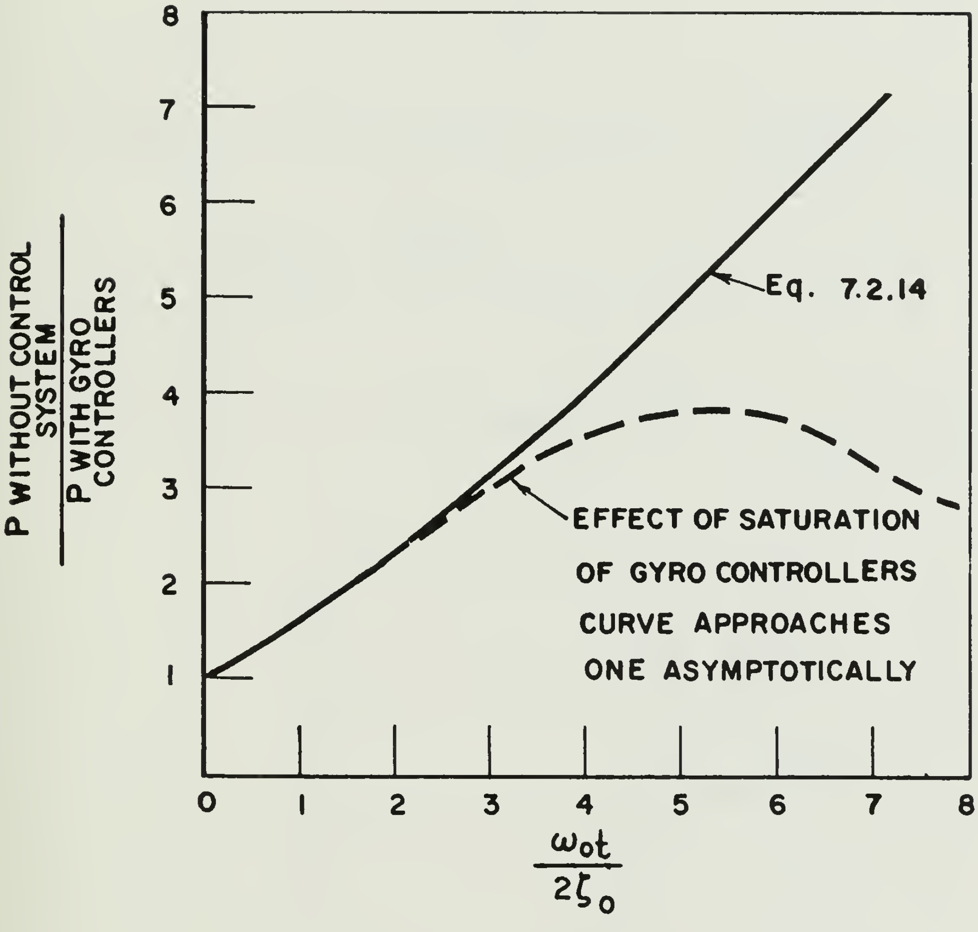


Figure 7. 2. 1 A plot of the ratio of roll rate of an uncontrolled spacecraft compared to a gyro controlled spacecraft both of which experience a step moment disturbance. Dotted curve is undocumented.

The solution for a step moment disturbance wherein the controller gimbal angles move through their complete range is solved by a phase plane analysis, however, first consider the special case of recovery from a large roll rate in the absence of external moments. The solution for the gimbal angle has been determined as follows.

$$\gamma_1 = \arc\tan \left\{ e^{-\frac{\omega_o}{2\zeta_o} t} \tan \gamma_o \right\} \tag{Eq. 7.2.15}$$

Using this relation for the gimbal angle gives the following equation for the roll rate.

$$p = p_{max} \sin \left[\arc\tan \left\{ e^{-\frac{\omega_o}{2\zeta_o} t} \tan \gamma_o \right\} \right] \tag{Eq. 7.2.16}$$

These equations are plotted in Figure 7. 2. 3 for an initial gimbal angle of 89 degrees.

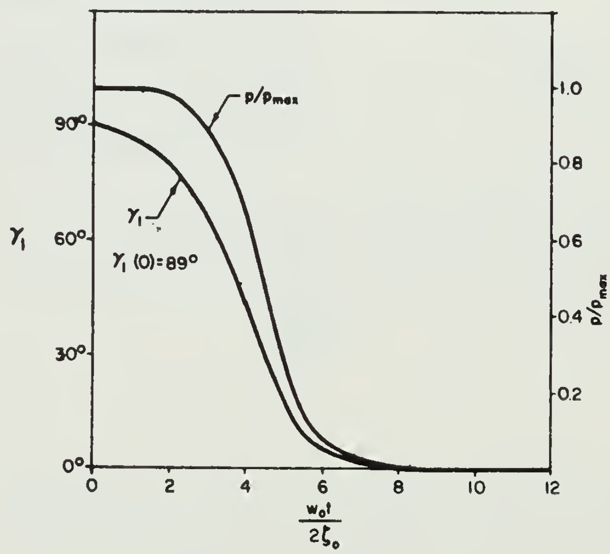


Figure 7. 2. 3 Plot of gimbal angle, γ_1, and non-dimensional roll rate, p/p_{max}, for spacecraft controlled by two gyro controllers with initial condition of $\gamma_1(0)$ = 89 degrees. Total angular momentum of system is zero and there are no external moments.

This special case of recovery from a large roll rate in the absence of external moments also has a simple phase plane solution as follows. As shown in Figure 7. 2. 4, plot the following equations for selected values of gimbal angle.

$$\dot{p} + \frac{(2\,H\,C\gamma_1)^2}{I_x\;k}\;p = 0 \qquad \text{(Eq. 7. 2. 17)}$$

$$p = p_{max}\,S\gamma_1 \qquad \text{(Eq. 7. 2. 18)}$$

The trajectory is determined by intersections of lines of the same gimbal angle. The times of the trajectory can be found from the ratio $\Delta p / \dot{p}_{average}$, and from this the curve roll rate versus time can be plotted. Thereafter, roll rate can be integrated to find the roll displacement as follows.

$$\phi = \int_{\phi(0)}^{\phi} p\,dt \qquad \text{(Eq. 7. 2. 19)}$$

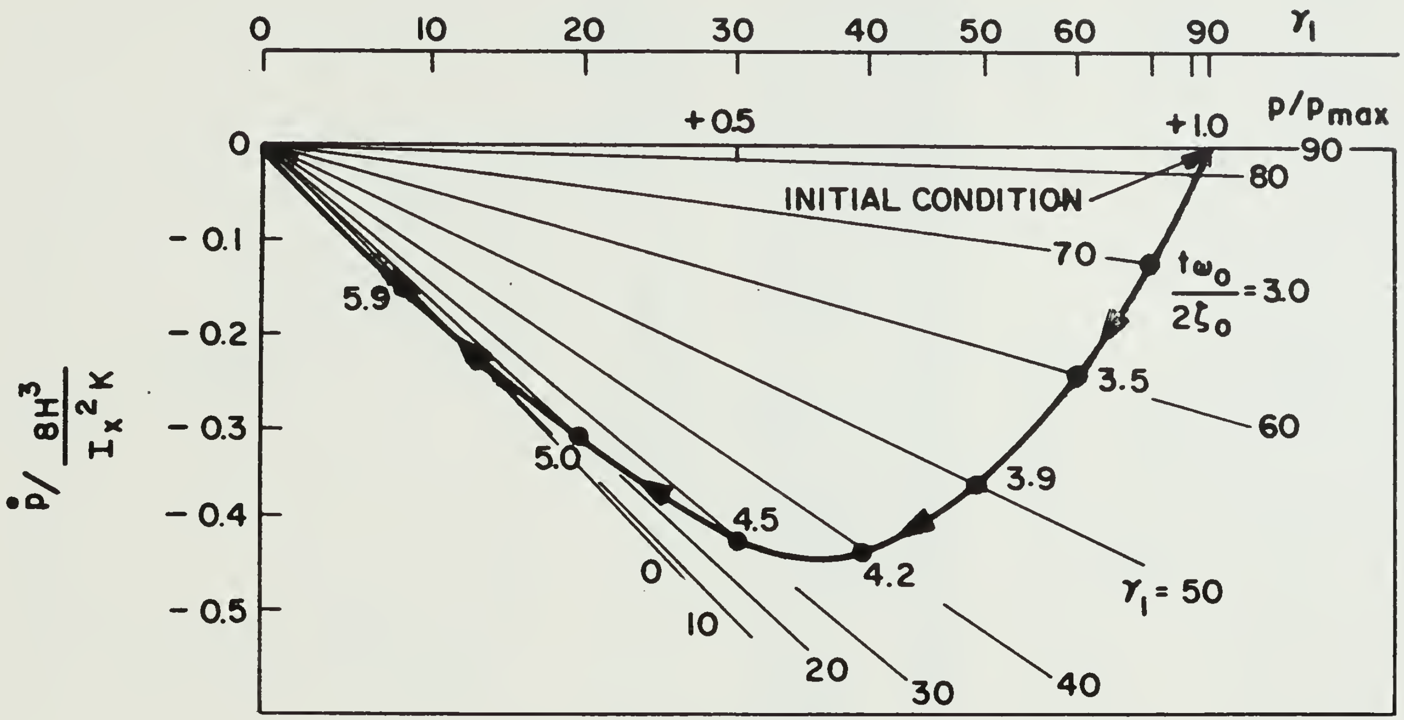


Figure 7.2.4 Phase plane trajectory of a spacecraft recovering from a maximum roll rate while controlled by a pair of gyro controllers with a zero input signal.

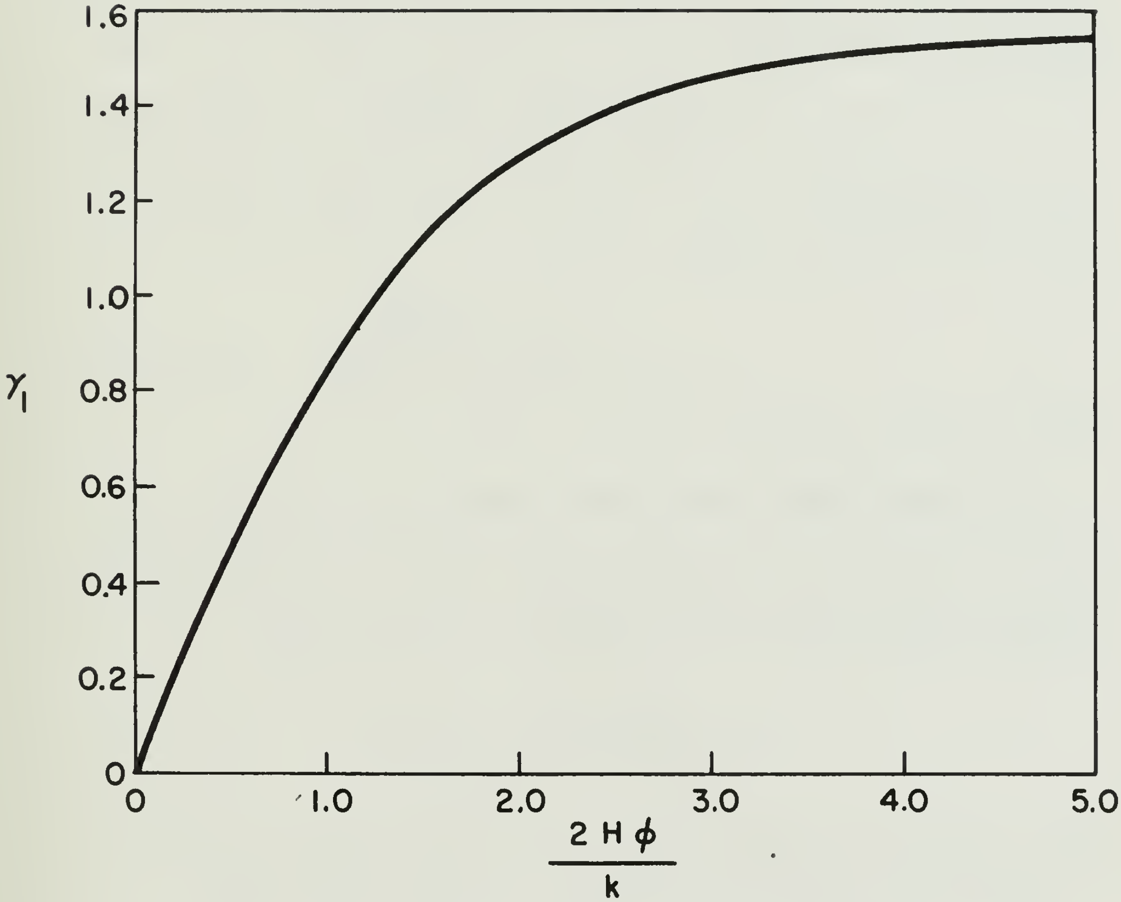


Figure 7.2.5 The relationship between gimbal angle and roll displacement for the zero input mode with zero initial conditions.

A closed form solution of equation 7. 2. 1 is difficult even when only the first order dynamics are considered. There is, however, a direct relationship between the vehicle attitude angular displacement and the gimbal angle. For example, the following equation is always true for zero initial conditions and for zero input to the controller.

$$J \ddot{\gamma}_1 + k \dot{\gamma} = -2 H C \gamma p \qquad \text{(Eq. 7. 2. 20)}$$

Considering only first order dynamics the equation can be integrated to give the following result.

$$\gamma = 2 \arctan \left\{ e^{-\frac{2 H \phi}{k}} \right\} - \frac{\pi}{2} \qquad \text{(Eq. 7. 2. 21)}$$

This equation is plotted in Figure 7. 2. 5.

An approximate phase plane analysis can be made of the position input mode operating to minimize a step torque disturbance by assuming that $\dot{\gamma}$ is constant. The first order differential equation approximation of equation 7. 2. 1 is given by

$$\dot{p} + \frac{4 H^2 (C\gamma_1)^2}{I_x k} p = \frac{M_x}{I_x} \qquad \text{(Eq. 7. 2. 22)}$$

This equation is plotted in Figure 7. 2. 6 for various contours of γ_1 which also represent contours of constant time since

$$\Delta t = \frac{\Delta \gamma_1}{\dot{\gamma}_1} \qquad \text{(Eq. 7. 2. 23)}$$

Since increased gimbal angles demand increased values of the roll rate, p, it is clear that $\dot{p}$ must also increase. At the end of the transient when $p = M_x k / 4 H^2$, $\dot{p}$ is very small, but its average value over the interval from $\gamma_1 = 0^o$ to $\gamma_1 = -10^o$ can be found by reading from Figure 7. 2. 6 the value of

$$\frac{\dfrac{\Delta p}{M_x k}}{4 H^2}$$

and computing $\dot{p}_{ave}$ from the relation

$$\dot{p}_{ave} = \frac{\Delta p}{\Delta t} \qquad \text{(Eq. 7.2.24)}$$

Figure 7. 2. 7 is an analog simulation of the zero input mode and shows that $\dot{\gamma}$ holds constant at $-M/2H$ until the gimbal angle gets beyond about 40 degrees. A phase trajectory determined by the method outlined above is shown in Figure 7. 2. 6.

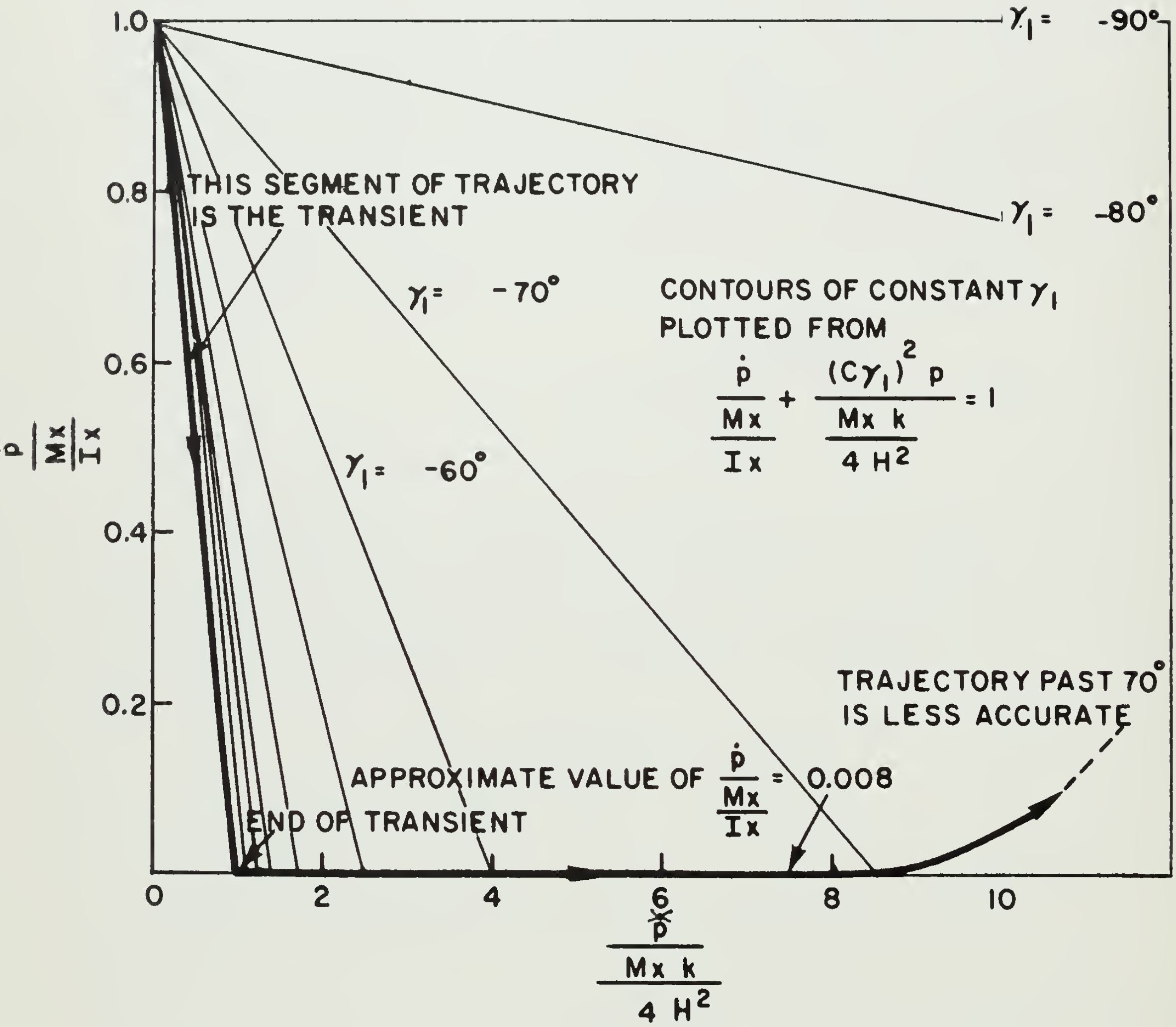


Figure 7.2.6 Estimated Phase Trajectory for Zero Input Mode Response to a Step Moment Disturbance.

All of the equations for zero input control have been presented in parametric form with no actual numerical values substituted in the equations. Therefore, the equations are valid for any general twin gyro controller system where interaxial coupling is negligible. The zero input mode is purely a rate stabilization system. It has been shown that an impulsive torque disturbance acting on the vehicle will cause the spacecraft to roll through a displacement angle, but that the roll rate will damp to zero. A step moment disturbance will cause a steady state roll and consequently an increasing roll displacement. The zero input system is primarily a rate stabilization system and the parameter which can conveniently be chosen to give the desired response is the damping coefficient, k, of the gyro controller. It is not necessary at this point to choose a particular value of k other than to say that this quantity is chosen on the basis of equation 7. 2. 6 which solved for the damping coefficient is written as follows.

$$k = 4 H \zeta_o \sqrt{\frac{J}{I_x}} \qquad \text{(Eq. 7. 2. 27)}$$

From a practical standpoint all of the quantities in equation 7. 2. 22 except ζ_o are dictated in the design by other considerations. I_x concerns the overall vehicle design and is determined by the mission and the detail design of the spacecraft. The angular momentum of the controller, H, is determined from controllability requirements, and the combined inertia of the gimbal, case, and rotor, J, is to be determined by optimum design of the controller to minimize the ratio of the total mass of the controller compared with the angular momentum of the controller. This leaves the parameter ζ_o which is to be chosen by the desired location of the closed loop poles of the zero input mode. Although the damping coefficient, k, is chosen primarily on the basis of the desired response to disturbances of the zero input mode, the choice does affect the other modes. For example, the closed loop poles of the zero input mode will be the open loop poles of the position

control mode. Therefore, let us delay the numerical choice of the damping ratio, ζ_o, until the equations of the other modes have been examined.

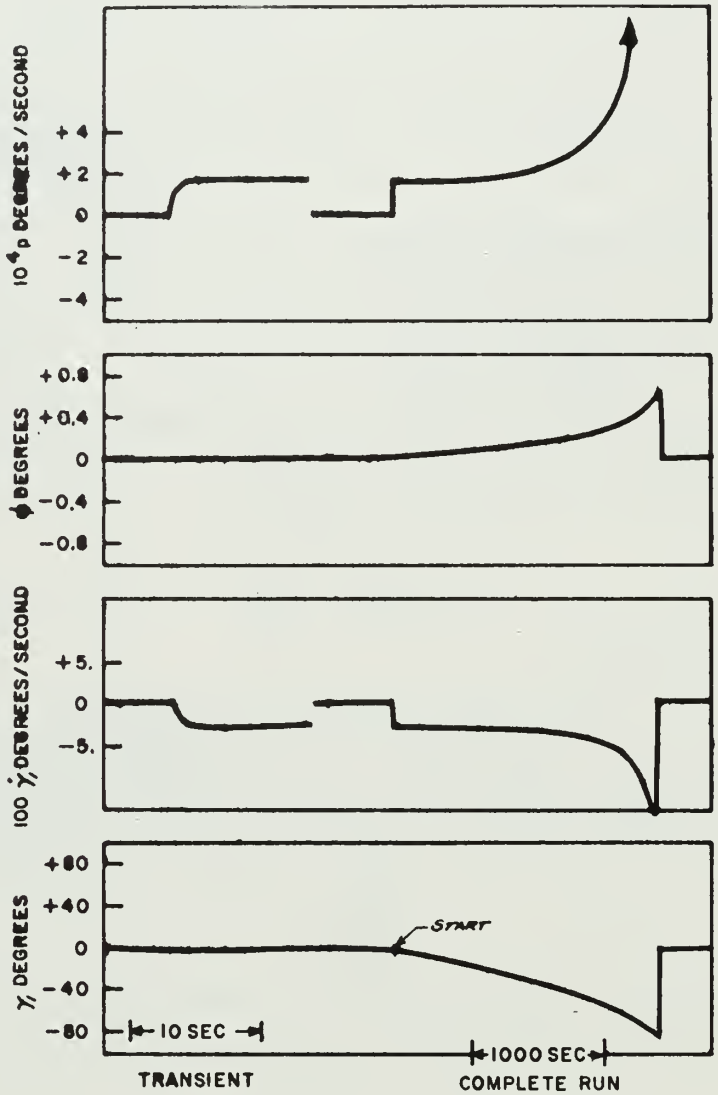


Figure 7.2.7 Spacecraft Response in Roll to a Step Moment Disturbance of 10 lb-ft. $I_x = 10^6$ lb-ft-sec^2, $J = 10$ lb-ft-sec^2 $H = 10^4$ lb.-ft-sec, $\zeta_o = 0.866$. Data from Analog Simulation.

7.3 Rate Control Mode

The equations for rate input control given by Equation 6.9.9 can be solved graphically by a plot similar to 7.2.4 where the trajectory is defined by the intersections of lines of equal gimbal angle. For this solution equation 6.9.9 is written in a slightly different form which, neglecting the external moment disturbance, is as follows.

$$\dot{p} + \frac{(2\,H\,C\gamma)^2}{I_x\;k}\,p = \left(\gamma_{1r} - \gamma_1\right)\frac{K_1\;\;2\,H\,C\gamma}{I_x\;k}$$

(Eq. 7.3.1)

This equation is plotted for p versus p in Figure 7.3.1 and gives contours of constant gimbal angle which slope from left to right. The second equation needed is that of equation 7.2.18 which is the requirement that angular momentum is conserved. This equation is plotted in Figure 7.3.1 as vertical contours of constant gimbal angle. The intersections provide the solution. The gain of the gimbal position control loop, K_1, has been allowed to have a relation to other parameters of the vehicle as follows.

$$K_1 = \frac{m\,4\,H^2}{I_x}$$

(Eq. 7.3.2)

The factor m may be called a gain ratio, and Figure 7.3.1 indicates that for high gain ratios the roll rate is an exponential represented by a constant slope on the plot of $\dot{p}$ versus p. The effect of the gain ratio factor, m, is shown more clearly in Figure 7.3.2 which is a plot of the steady state gimbal angle versus the value of the step input. This plot shows that for very high gain the gimbal angle is linear with the input angle, but for a gain ratio of one the variation is highly non-linear. If the gain ratio is less than one, the position control system does not have the capability of holding the complete range of gimbal angles in a stable manner. Figures 7.3.3 and 7.3.4 further illustrate the effect of low gain ratios. Figure 7.3.3 shows a plot of steady

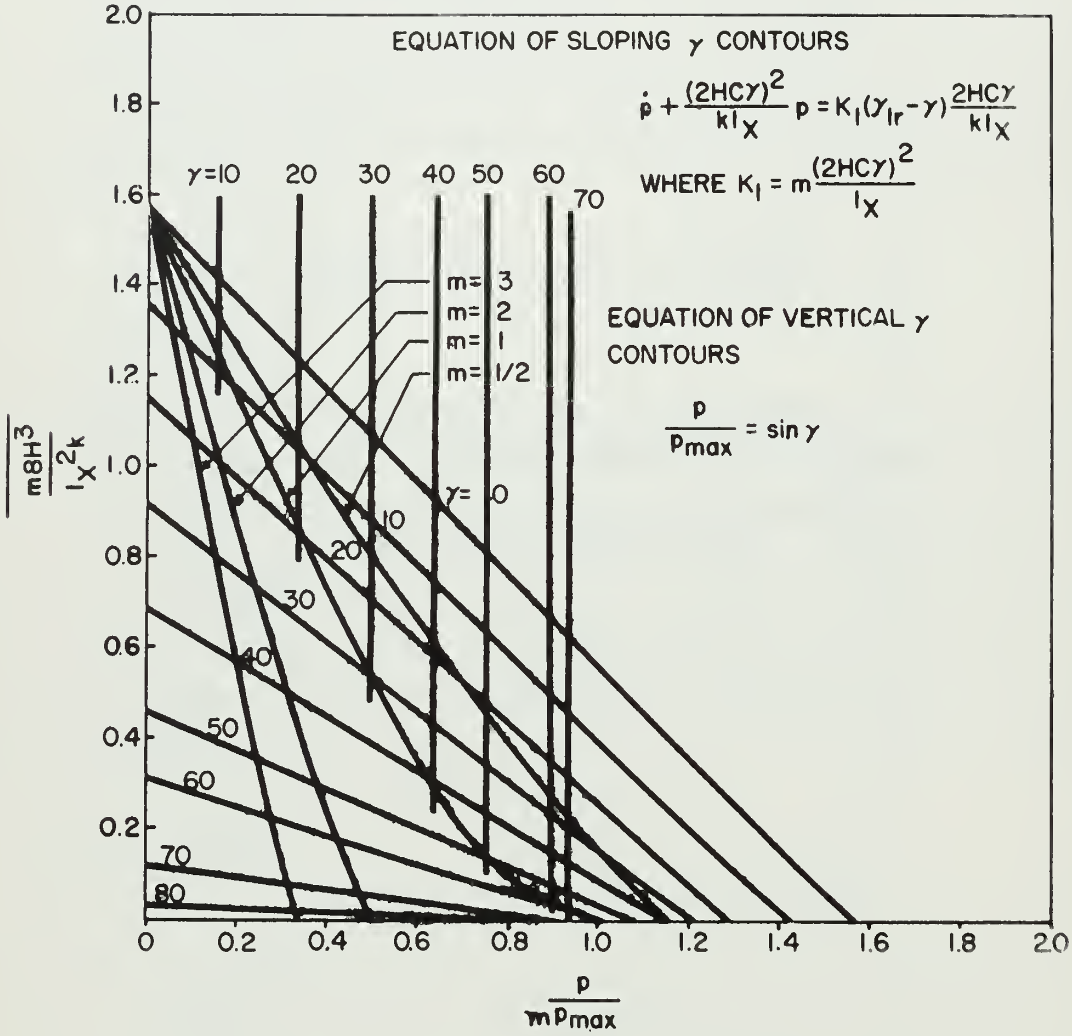


Figure 7.3.1 Phase trajectory for spacecraft response to a full step displacement rate control input. The external moment disturbance is zero. The response is defined by the intersections of the lines of constant gimbal angles plotted for the two equations of motion shown on the figure.

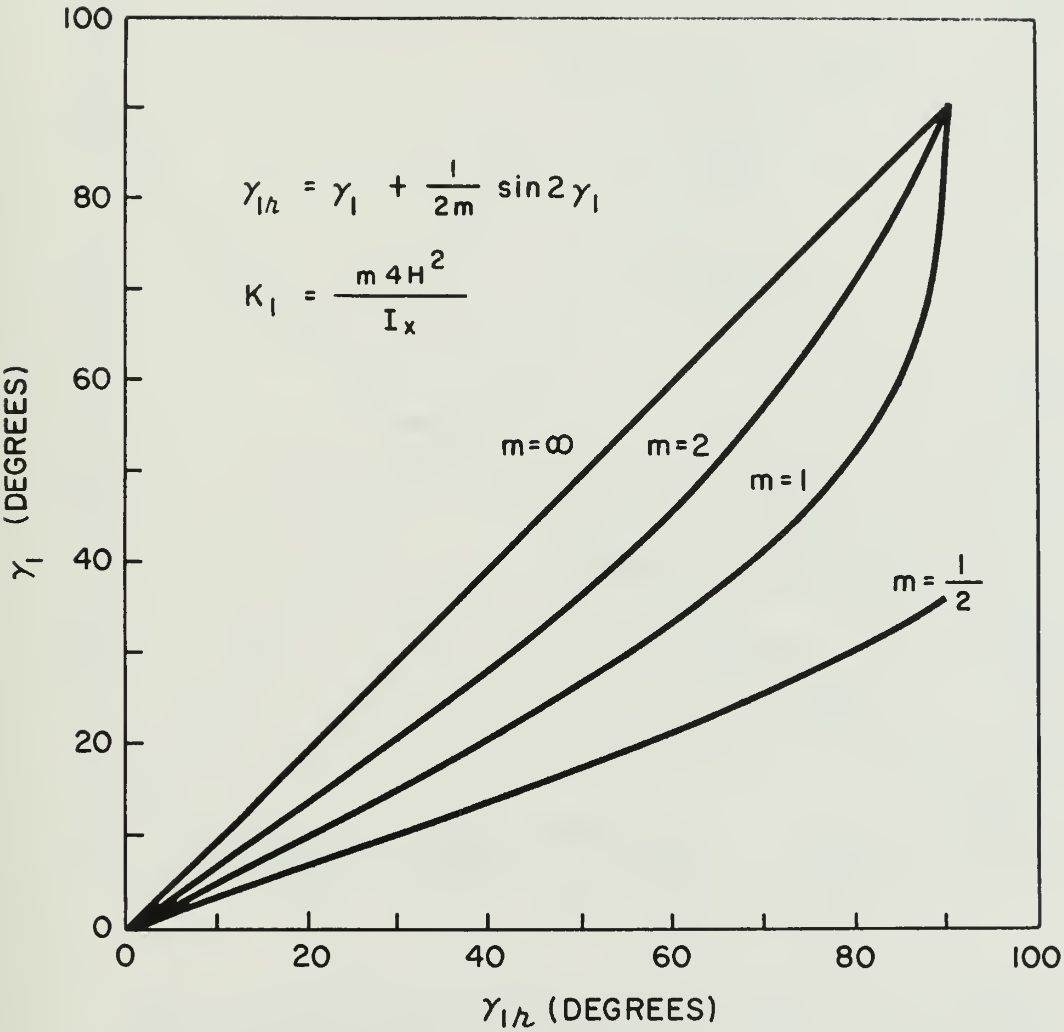


Figure 7.3.2 Plot of steady state gimbal angle, γ_1, resulting from step input, γ_{1r}, for rate input mode.

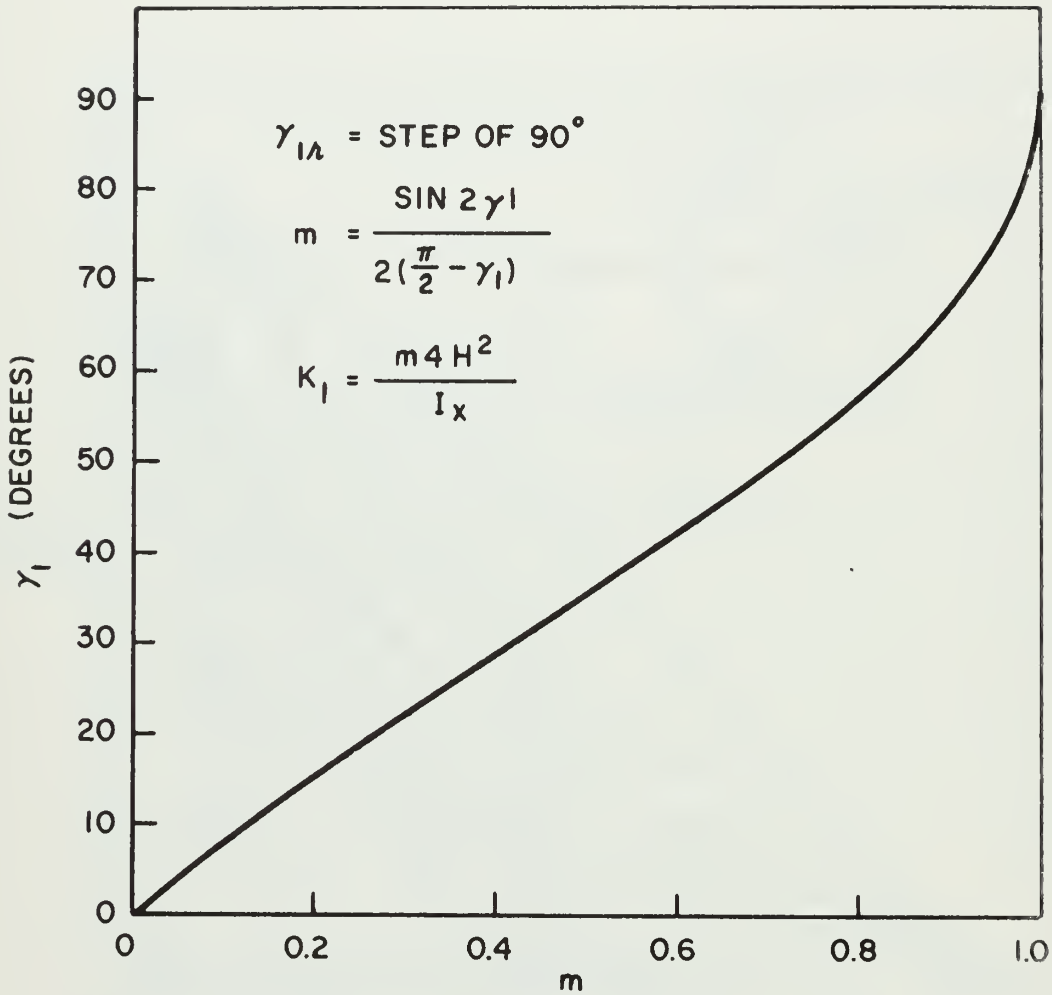


Figure 7.3.3 Steady state values of roll gimbal angle for gain ratios less than one in the gimbal angle position control loop used in rate input control.

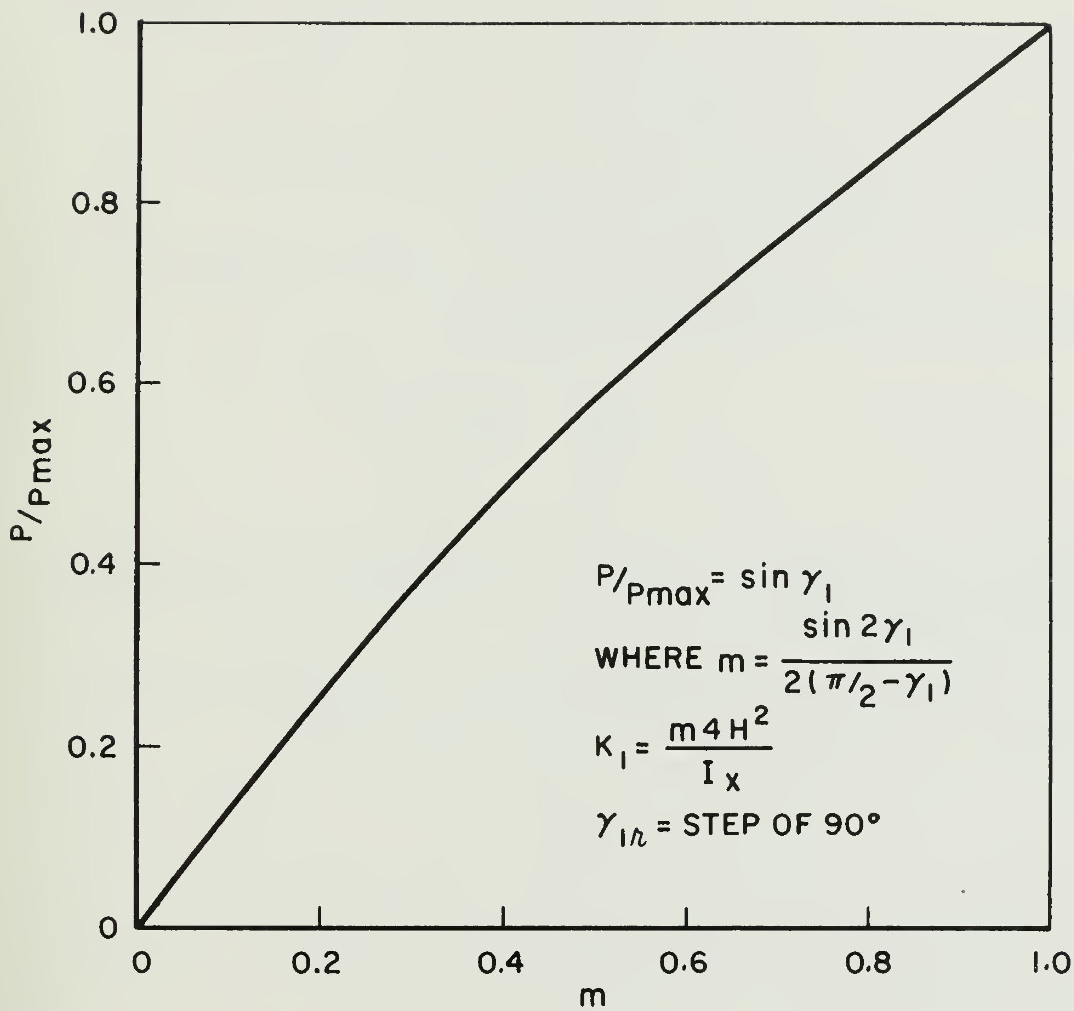


Figure 7.3.4 Plot of roll capability for gain ratios less than one in the gimbal angle position control loop used in rate input control mode.

state gimbal angle for a step input of ninety degrees versus the full range of gain ratios less than one. Careful inspection of the curvature shows an inflection point at 45 degrees which was shown to be a peak control power point in section 5. 31. Figure 7. 3. 4 shows the roll capability for a step input of ninety degrees for the range of gain ratios less than one.

From a stability standpoint the rate input mode has a natural frequency from equation 6. 9. 6 equal to

$$\omega_n = \sqrt{\frac{K_1}{J} + \frac{\left(2 H C\gamma_1\right)^2}{I_x J}} \qquad \text{(Eq. 7. 3. 3)}$$

This natural frequency is greater than that for zero input control given by equation 7. 2. 5.

The product of the damping ratio and the natural frequency are the same for zero input and rate input. Thus

$$2 \zeta_r \omega_n = \frac{k}{J} \qquad \text{(Eq. 7. 3. 4)}$$

If the damping coefficient, k, is chosen on the basis of zero input control then according to equation 7. 2. 6,

$$k = 4 H \zeta_o \sqrt{\frac{J}{I_x}} \qquad \text{(Eq. 7. 3. 5)}$$

Substituting this k into equation 7. 3. 4 gives

$$2 \zeta_r \omega_n = \frac{4 H \zeta_o}{\sqrt{I_x J}} \qquad \text{(Eq. 7. 3. 6)}$$

Then further substitution of K_1 in equation 7. 3. 2 into equation 7. 3. 3 gives

$$\omega_n = \frac{2 H}{\sqrt{I_x J}} \sqrt{m + (C\gamma)^2} \qquad \text{(Eq. 7. 3. 7)}$$

Therefore,

$$\zeta_r = \frac{\zeta_o}{\sqrt{m + (C\gamma)^2}} \qquad \text{(Eq. 7.3.8)}$$

which is to say that the damping ratio for rate control is proportional to the damping ratio for zero input control by the factor $1/\sqrt{m + (C\gamma)^2}$. The greatest difference between the two damping ratios occurs at zero gimbal angle. Hence,

$$\zeta_r = \frac{\zeta_o}{\sqrt{m + 1}} \qquad \text{(Eq. 7.3.9)}$$

Actually, large values of the gain factor m gives more precise positioning of the gimbal angle as shown by Figure 7.3.2, but the greater this gain the more lightly damped (more oscillatory) becomes the rate control mode. It appears that an acceptable solution lies in the choice of the gain ratio, m, as unity. For a gain ratio of unity the gimbal angle can be positioned at the full ninety degree position, yet for the range of 45 degrees or less the curve for m = 1 of Figure 7.3.2 is approximately linear. A gimbal angle of 45 degrees gives more than 70 of the maximum roll rate; therefore, it is not likely that the vehicle will be operated at angles of more than 45 degrees except for the full roll rate position of 90 degrees. For unity gain ratio

$$\zeta_r = \frac{\zeta_o}{\sqrt{2}} \qquad \text{(Eq. 7.3.10)}$$

By cross-plotting the steady state response to an impulsive disturbance torque on the spacecraft shown in Figure 7.3.5 the roll rates can be compared with those for zero input mode and those for no control system aboard the vehicle. For unity gain the rates are approximately one-half that of a vehicle without a control system.

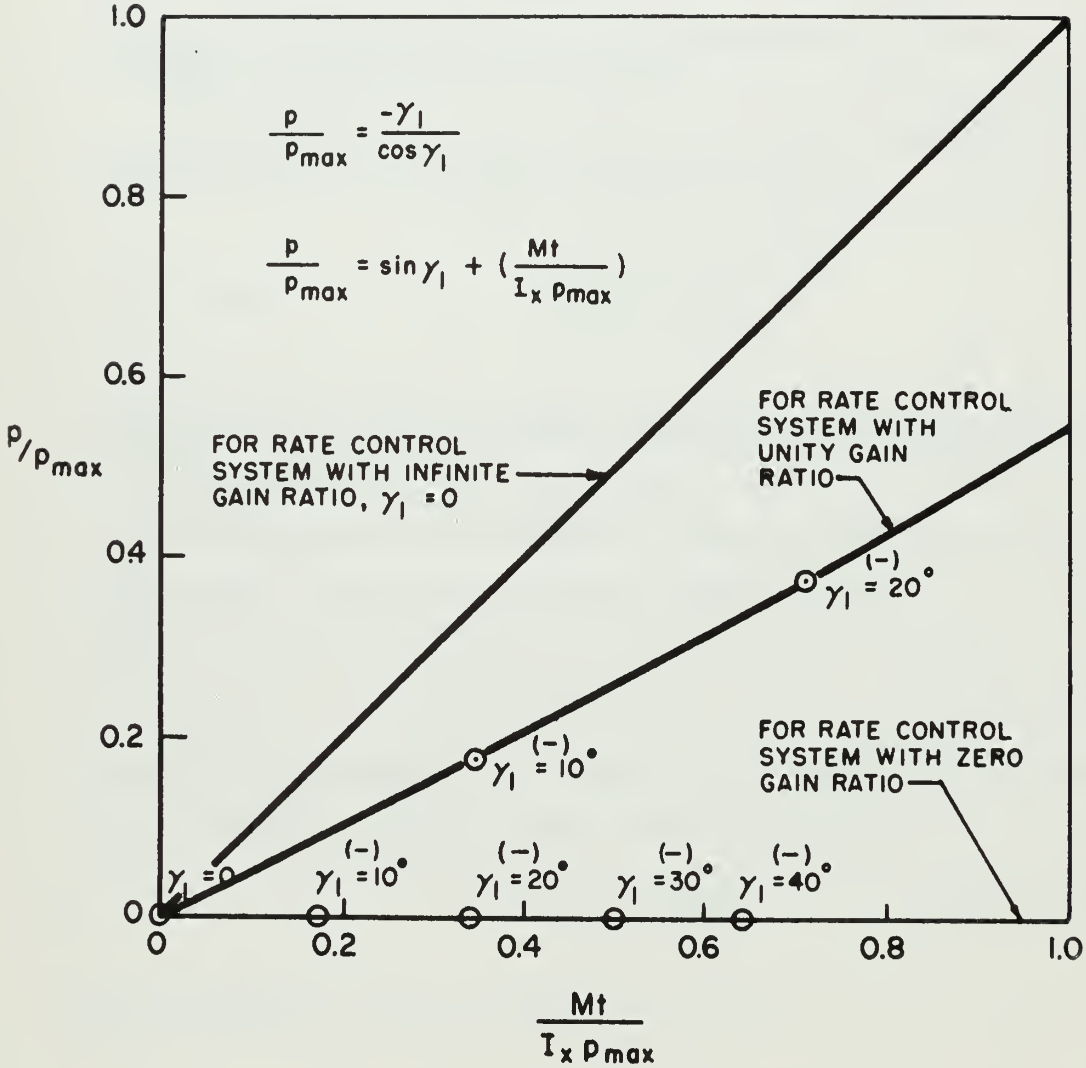


Figure 7.3.5 Plot of steady state roll rate versus magnitude of torque disturbance impulse for rate control mode. Infinite gain ratio corresponds to controllers that are held rigidly fixed in the vehicle, and is the same as vehicle response with out a control system. Zero gain ratio corresponds to zero input mode of section 7.2.

7.4 Position Control Mode

The position control mode is formed by closing an attitude angle feedback loop around the zero input mode. A convenient method for studying the position control mode is a locus of roots which is a useful technique found in most control theory texts. Using root locus theory it is known that the closed loop poles of the zero input mode form the open loop poles of the position control mode. Therefore, the open loop equation for the position control mode can be written for roll motion as

$$(\phi)_{\substack{\text{open}\\ \text{loop}}} = \frac{\left(K_x 2 H C\gamma_1\right) \phi_r + F_{1\gamma} M_x}{S\left(I_x F_{1\gamma} S + 4 H^2 \left(C\gamma_1\right)^2\right)} \qquad \text{(Eq. 7.4.1)}$$

where the gimbal angle must be considered approximately constant. The closed loop equation is given by

$$\phi = \frac{\left(K_x 2H C\gamma_1\right) \phi_r + F_{1\gamma} M_x}{\left(I_x F_{1\gamma} S^2 + 4 H^2 \left(C\gamma_1\right)^2 S + K_x 2 H C\gamma_1\right)} \qquad \text{(Eq. 7.4.2)}$$

The poles of the denominator of equation 7.4.1 are to be determined so that the position control mode has good response to input commands and to minimize the disturbances of external torques on the spacecraft. An interesting theorem which applies to a control system with no zeros and three open loop poles, one of which is at the origin, is as follows: the natural frequency of a unity feedback control system having no zeros and three open loop poles, one of which is at the origin, can never be greater than the natural frequency of the system operating open loop. This theorem is easily proven by substituting $j\omega$ in the characteristic equation of the closed loop system and equating the imaginary part to zero. The theorem is illustrated in Figure 7.4.1 by noting that all stable poles of the closed loop system lie within the circle of radius ω_o

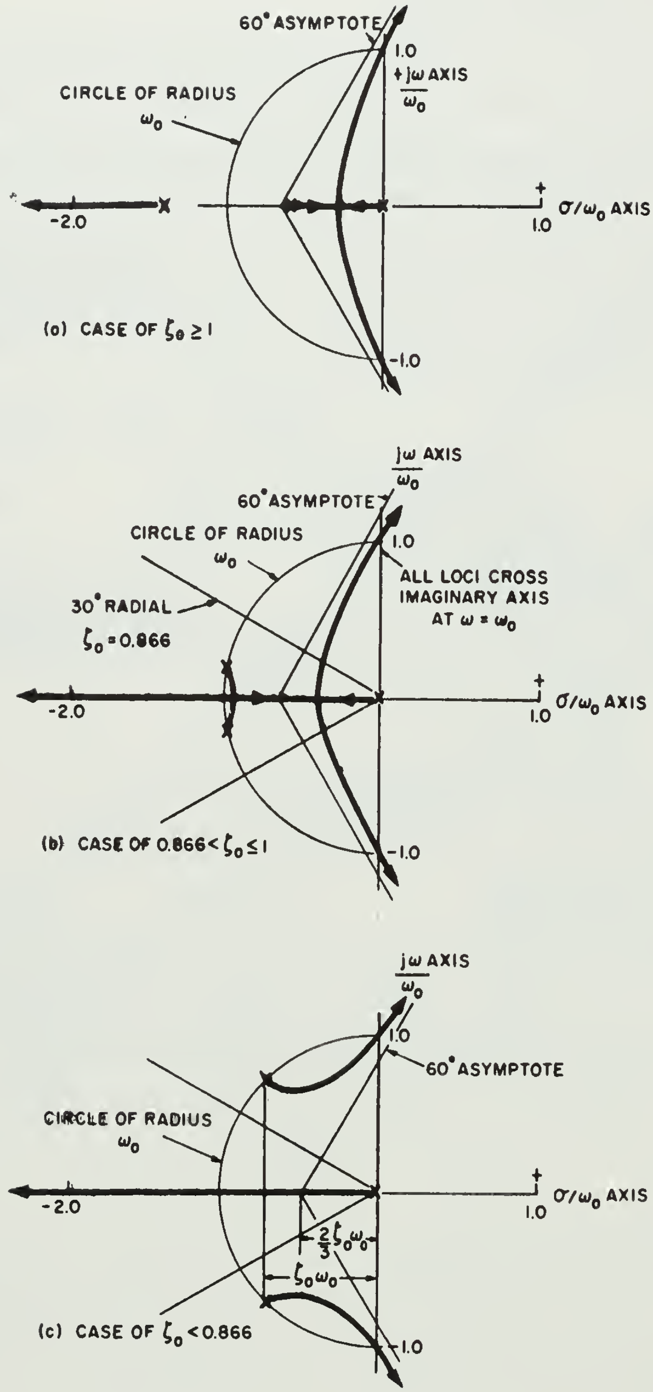

60° ASYMPTOTE
CIRCLE OF RADIUS ω_0
1.0
$+j\omega$ AXIS / ω_0
-2.0
1.0
σ/ω_0 AXIS
-1.0
(a) CASE OF $\zeta_0 \geq 1$
$j\omega$ AXIS / ω_0
60° ASYMPTOTE
CIRCLE OF RADIUS ω_0
1.0
ALL LOCI CROSS IMAGINARY AXIS AT $\omega = \omega_0$
30° RADIAL
$\zeta_0 = 0.866$
-2.0
1.0
σ/ω_0 AXIS
-1.0
(b) CASE OF $0.866 < \zeta_0 \leq 1$
$j\omega$ AXIS / ω_0
1.0
60° ASYMPTOTE
CIRCLE OF RADIUS ω_0
-2.0
1.0
σ/ω_0 AXIS
$\frac{2}{3}\zeta_0\omega_0$
$\zeta_0\omega_0$
(c) CASE OF $\zeta_0 < 0.866$
-1.0

Figure 7.4.1

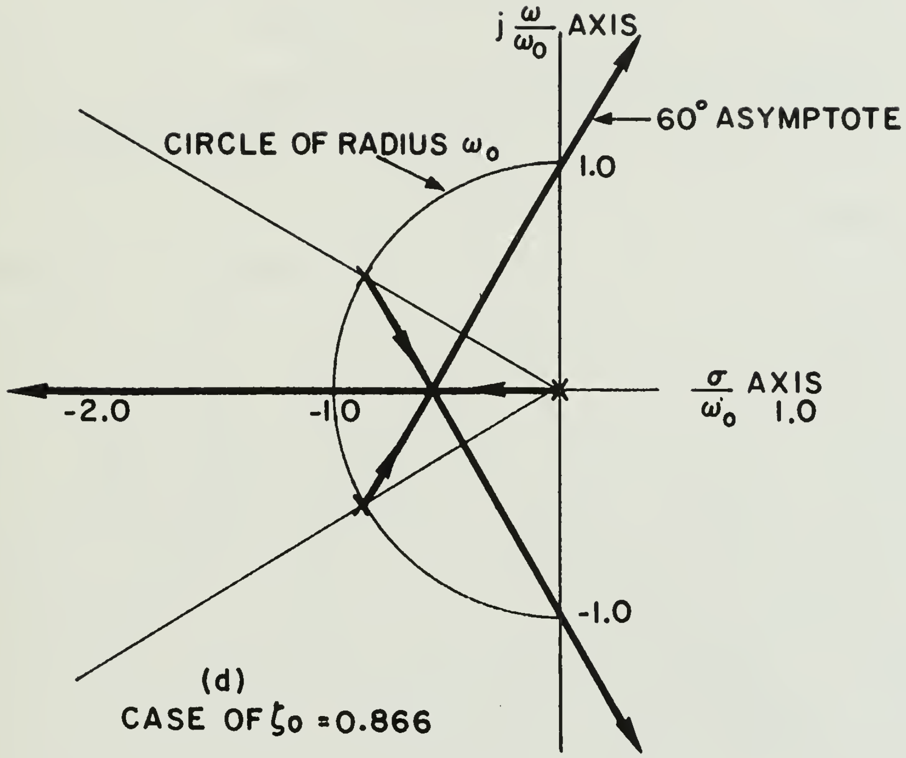


Figure 7.4.1 Illustration of four possible geometric patterns for the locus of roots for the position control mode. The three poles shown are the poles of the closed loop zero input mode which constitute the open loop poles of the position control mode. The closed loop poles of the position control mode will lie on the root loci depending on the value of the gain of the position control loop.

which is the natural frequency of the open loop system. The primary purpose in Figure 7.4.1 is to illustrate the four possible geometric patterns that the locus of roots of the position control system can have. If the damping ratio of the zero input mode, ζ_o, is greater than one, then Figure 7.4.1 (a) applies. If the damping ratio is less than one but greater than 0.866 then Figure 7.4.1 (b) applies. If less than 0.866 then Figure 7.4.1 (c) represents the locus of roots. The case where ζ_o is exactly equal to 0.866 is shown in Figure 7.4.1 (d). The overdamped case (a) has poor response at low gains, and it becomes too oscillatory at the higher gains. Values of ζ_o less than 0.7 are not satisfactory because the rate control mode and the position control mode both become too oscillatory. Values of ζ_o between 0.7 and 1.0 give an acceptable solution so that the vehicle has good response to input commands as well as good response to damp torque disturbances.

As an example of this section let us solve the equations of motion using a damping ratio of $\zeta_o = 0.866$ such that the locus of roots of Figure 7.4.1 (d) applies. This is not suggested as an optimum choice but is considered to be an example of a satisfactory location of the closed loop poles. Choosing a value of K_x such that all three closed loop poles coalesce gives the following equations.

$$K_x = 0.77 \frac{H^2}{\sqrt{I_x J}} \qquad \text{(Eq. 7.4.3)}$$

$$\phi = \frac{0.192\,\omega_o^3\, C\gamma_1\, \phi_r + \left(S + 3\left(0.577\,\omega_o\right)\right) M_x/I_x}{S^3 + 3\left(0.577\,\omega_o\right)S^2 + 3\left(0.577\,\omega_o\right)^2\left(C\gamma_1\right)^2 S + 0.192\,\omega_o^3\, C\gamma_1}$$

(Eq. 7.4.4)

where ω_o is natural frequency of zero input mode given by equation 7.2.7.

To further explain equation 7.4.4, if the cubic denominator is written in the following form

$$\left(S^2 + 2\zeta_\rho \omega_n S + \omega_n^2\right)\left(S + \xi\right) = 0 \qquad \text{(Eq. 7.4.5)}$$

then the solution chosen as an example has the following values for the parameters.

$$\zeta_\rho = 1 \qquad \text{(Eq. 7.4.6)}$$

$$\omega_n = 0.577\ \omega_o \qquad \text{(Eq. 7.4.7)}$$

$$\xi = 0.577\ \omega_o \qquad \text{(Eq. 7.4.8)}$$

The primary purpose of the position control mode is to enable the spacecraft to track a reference line. This reference line may be the line of sight to a star, and usually in this mode the vehicle attitude rates are either small or are constant. It is considered that equation 7.4.4 can be evaluated for constant gimbal angles if we assume that the error signal from the sensor is limited to some maximum value. In actual practice this limiting will be accomplished by the saturation of the sensor. If the maximum error signal from the sensor is ϕ_e then the steady state roll rate resulting from the constant sensor error can be derived as the following equation.

$$\phi_e = 5.2\sqrt{\frac{J}{I_x}}\left[\frac{p}{p_{max}}\right]\left[1-\left(\frac{p}{p_{max}}\right)^2\right]^{1/2} \qquad \text{(Eq. 7.4.9)}$$

This curve is shown in Figure 7.4.2 and shows that for a given error signal, ϕ_e, there are two solutions for roll rate. The solution at the higher rate is unstable for the same reasons that the initial point at p_{max} is unstable in Figure 7.2.4. Therefore, to avoid all possibilities of the position control mode being unstable the error signal from the sensor is limited to that giving

a maximum gimbal angle of 30^o which allows the position control loop to drive the rate of the spacecraft up to one-half p_{max}. This limiting also enables the stability analysis of the position control equations to be accomplished with a constant gimbal angle. Thus for small gimbal angles the position control equation 7. 4. 4 can be written as follows.

$$\phi = \frac{0.192\,\omega_o^3\,\phi_r + \left[S + 3\left(0.577\,\omega_o\right)\right] M_x/I_x}{\left(S + 0.577\,\omega_o\right)^3}$$

(Eq. 7. 4. 10)

For zero sensor input the response to an impulsive moment disturbance is given by

$$\phi = \frac{M_o}{I_x}\left(0.577\,\omega_o\, t^2 + t\right) e^{-0.577\,\omega_o t}$$

(Eq. 7. 4. 11)

The response to a step moment disturbance is given by

$$\phi = \frac{M_1}{I_x}\left\{\frac{3\,u_1(t)}{\left(0.577\,\omega_o\right)^2} - \left[t^2 + \frac{3\,t}{\left(0.577\,\omega_o\right)} + \frac{3}{\left(0.577\,\omega_o\right)^2}\right] e^{-0.577\,\omega_o t}\right\}$$

(Eq. 7. 4. 12)

For zero moment disturbance the response to a step position input angle is given by

$$\phi = \phi_1\left\{u_1(t) - \left[\frac{\left(0.577\,\omega_o\right)^2}{2}\, t^2 + 0.577\,\omega_o t + 1\right] e^{-0.577\,\omega_o t}\right\}$$

(Eq. 7. 4. 13)

The response to an impulsive input is given by

$$\phi = \phi_o \left\{ \frac{(0.577\,\omega_o)^3\, t^2}{2}\; e^{-0.577\,\omega_o t} \right\} \qquad \text{(Eq. 7.4.14)}$$

The response is seen to be well behaved at small gimbal angles; however, consider the location of the poles of the closed loop position control mode as the gimbal angle becomes larger. Figure 7.4.3 shows the migration of the poles for the value of K_x chosen in equation 7.4.3. This figure indicates that at gimbal angles greater than 30 degrees the system becomes very oscillatory, and at approximately 84 degrees the system becomes unstable. The effect of the initial choice in open loop damping ratio, ζ_o, is shown in Figure 7.4.4 which is plotted for the same K_x. A smaller K_x would of course raise the curve of Figure 7.4.4 so that in the end a singular point at 90 degrees would result.

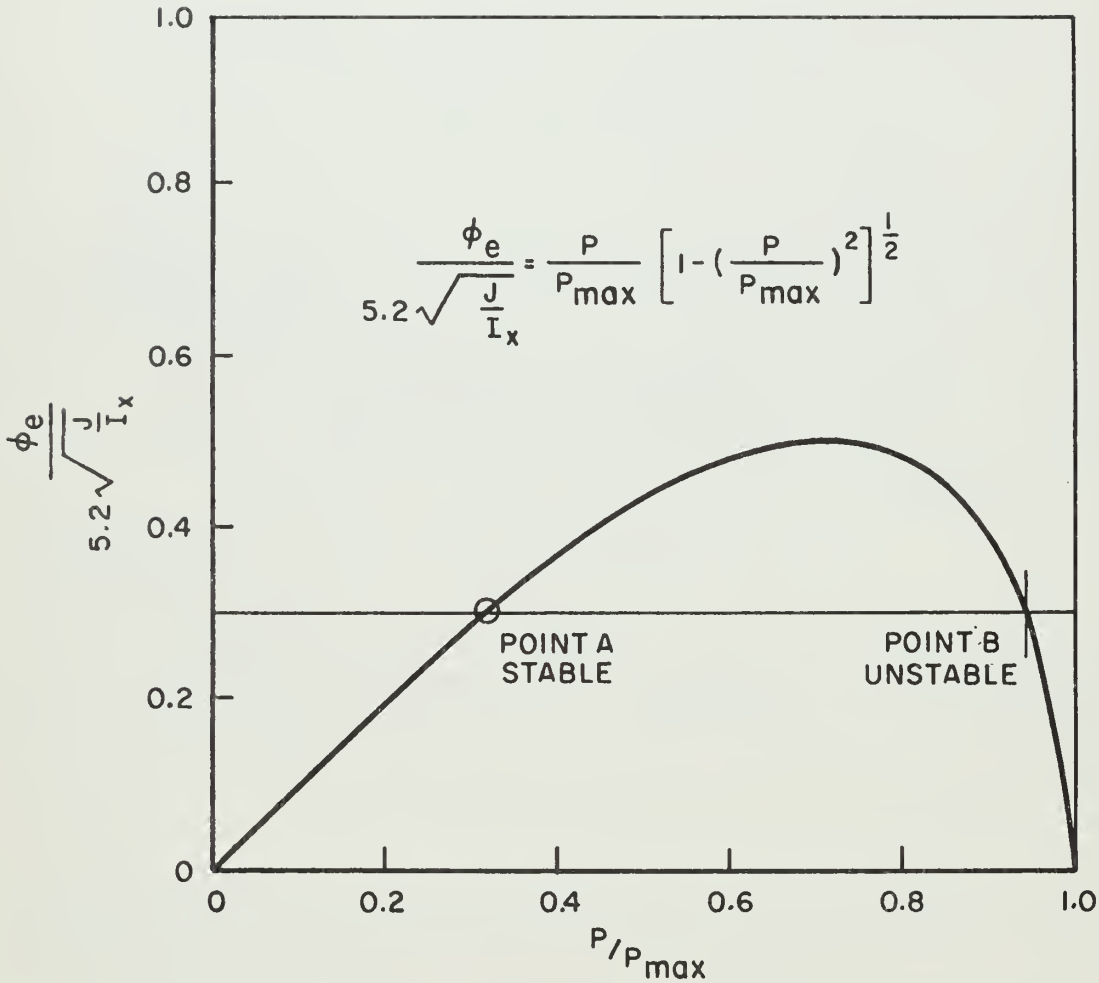


Figure 7.4.2 Steady state roll rate for constant sensor error signal, $\frac{\phi_e}{5.2\sqrt{J/I_x}}$ of 0.3 is an example only.

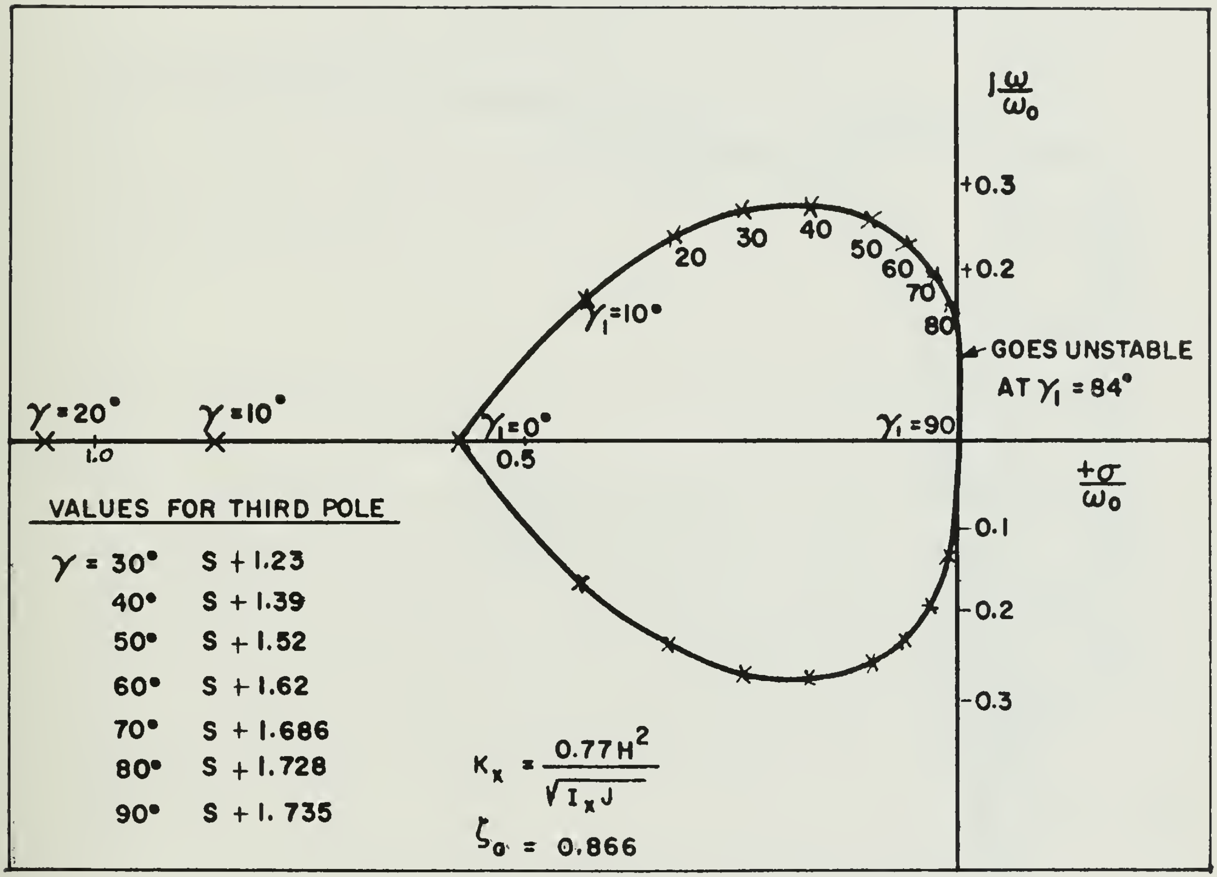


Figure 7.4.3 Location of Position Control Mode Closed Loop Poles for Large Gimbal Angles.

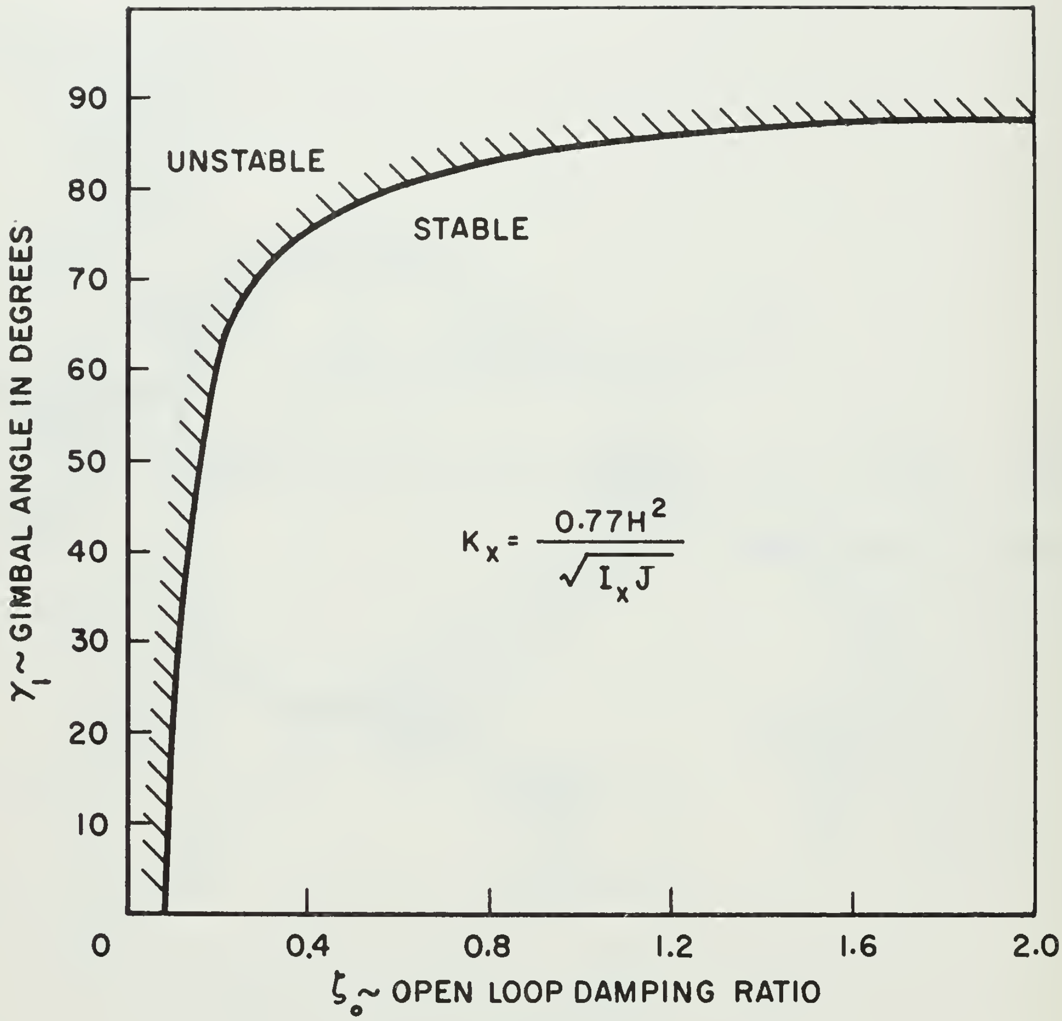


Figure 7.4.4 Plot of Boundary between Stable and Unstable Operation in Position Control Mode.

7.5 Adaptive Mode

It has been stated earlier that the four gyro system has the adaptive feature that permits continuous three axis control in the event that one of the four gyros is lost. This mode can be illustrated by assuming the loss of the number four gyro controller and examining the resulting spacecraft response. At the outset, let it be assumed that the controller fails such that it loses its angular momentum at a steady rate over time, t_1. Assuming no interaxial coupling this gives a moment disturbance about the roll axis as follows.

$$\frac{dH}{dt} = M_x \qquad \text{(Eq. 7.5.1)}$$

The moment disturbance is seen to be a step of H/t_1 which lasts for t_1 seconds, and this gives a total torque impulse of H.

The spacecraft response to such a disturbance can be obtained from the equations of the previous section; however to illustrate the response for the adaptive mode the problem was set up on an analog computer and the results are shown in Figure 7.5.1 and 7.5.2 for the case where a sensor is providing attitude errors. The zero input mode also operates to provide rate stabilization upon the loss of any controller.

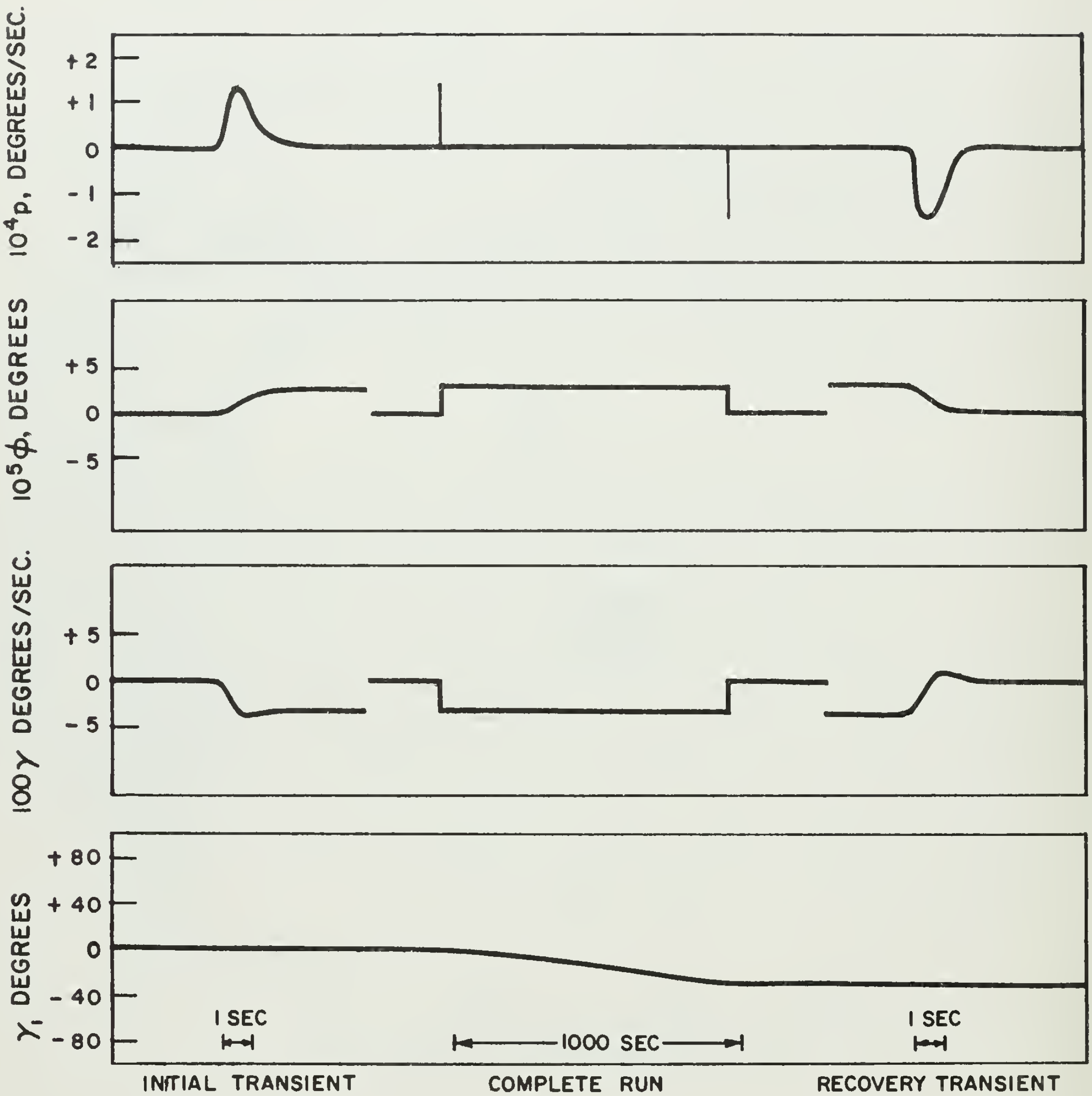


Figure 7.5.1 Spacecraft Roll Rate and Roll Angle with Gimbal Rate and Angle for the Adaptive Mode. Data is from Analog Computer. $I_x = 10^6$ lb-ft-sec^2, $J = 10$ lb-ft-sec^2, $t_1 = 1000$ sec. $H = 10^4$ lb-ft-sec, $\zeta = 0.866$. See Figure 7.5.2 for Phase Trajectory. Complete Run includes Initial Transient and Recovery Transient.

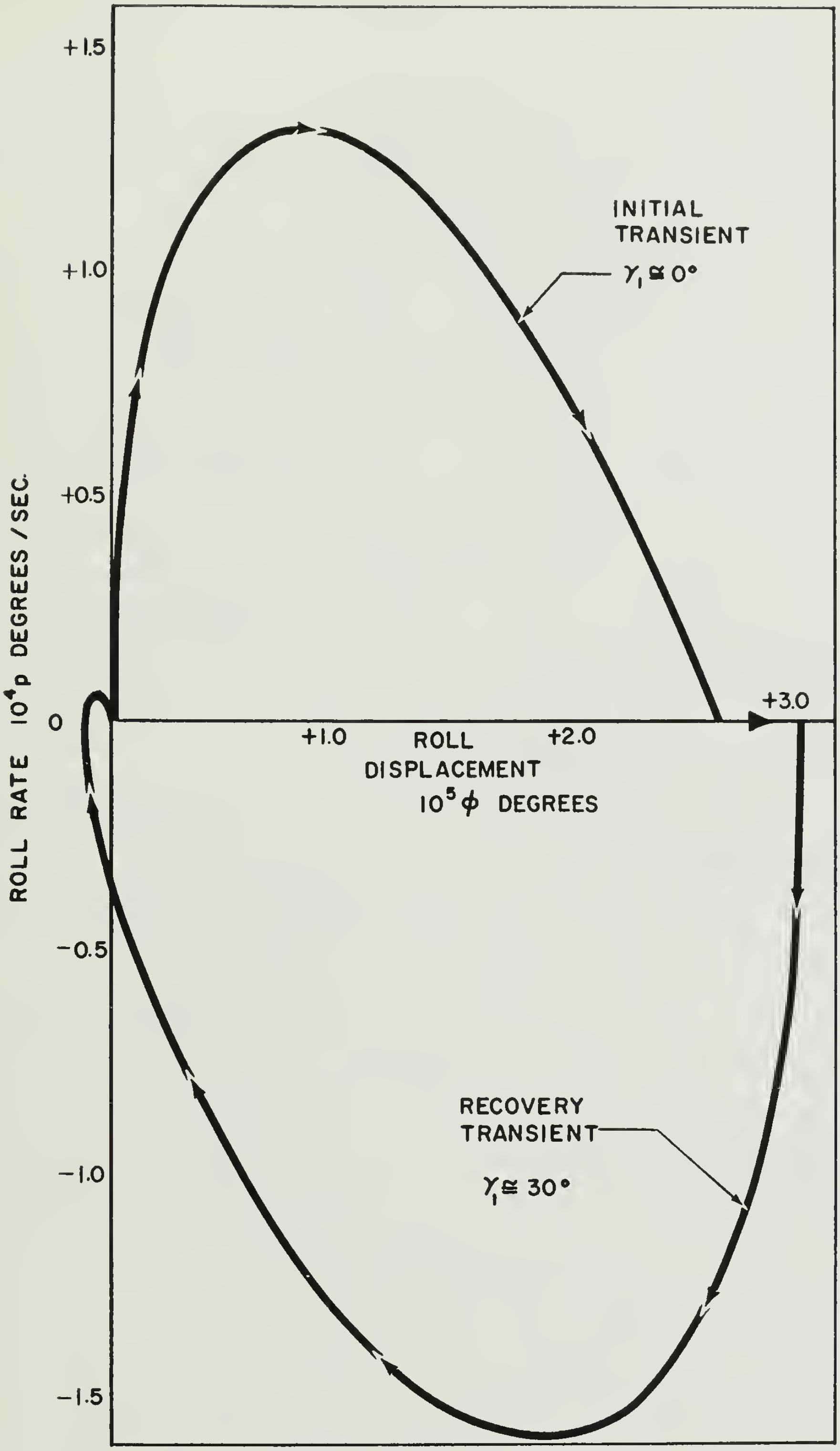


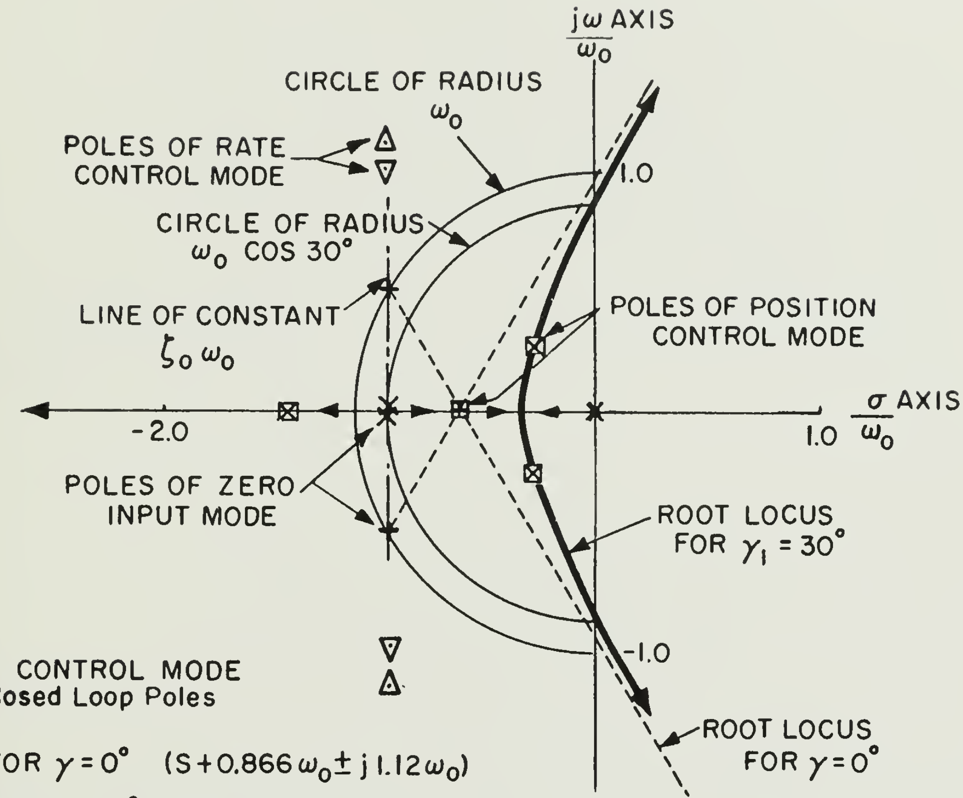
Figure 7.5.2 Phase Trajectory for Adaptive Mode
$I_x = 10^6$ lb-ft-sec^2, $J = 10$ lb-ft-sec^2, $H = 10^4$ lb-ft-sec,
$\zeta_o = 0.866$, See Figure 7.5.1.

7.6 Summary

This chapter has evaluated the response of the system to input disturbances and input commands for the zero input mode, the rate control mode, and the position control mode. The results are shown in the illustration of the location of the poles of the various modes shown in Figure 7.6.1. This plot shows that an initial choice of the damping ratio of the zero input mode, ζ_o, of approximately 0.866 gives satisfactory performance for the rate control mode and the position control mode; therefore, no adaptive type change in the damping of the gimbal or tandem compensation is required to satisfy all three modes of operation. The effects of increase in gimbal angle is seen to make the zero input mode and the rate control mode less oscillatory, whereas the position control mode becomes more oscillatory. For a fixed value of gain, K_x, there exists a gimbal angle for the position control mode which makes the system unstable.

The system operating in the position control mode has inherent adaptive characteristics in that the failure of any one of the gyro controllers will automatically be compensated for by the repositioning of the opposite pair of controllers.

The examples have been given for the roll equations. The pitch equations are almost identical except for the sign of the gain K_y. The general solution for the yaw equations are identical except that the functions $F_{1\gamma}$ and $F_{3\gamma}$ have been defined for a pair of controllers whereas G_α is defined for a single controller. Also, the yaw controller uses all four controllers, and therefore faster response is available, if desired.



RATE CONTROL MODE
Closed Loop Poles

Δ FOR $\gamma = 0°$ $(S + 0.866\,\omega_0 \pm j\,1.12\,\omega_0)$

∇ FOR $\gamma = 30°$ $(S + 0.866\,\omega_0 \pm j\,1.0\,\omega_0)$

POSITION CONTROL MODE
Open Loop Poles

\+ FOR $\gamma = 0°$ $S(S + 0.866\,\omega_0 \pm j\,0.5\,\omega_0)$

× FOR $\gamma = 30°$ $S(S + 0.866\,\omega_0)^2$

$K_x = 0.77 \frac{H^2}{\sqrt{I_x J}}$

Closed Loop Poles

⊞ FOR $\gamma = 0°$ $(S + 0.577\,\omega_0)^3$

⊠ FOR $\gamma = 30°$ $(S + 1.23\,\omega_0)(S + 0.25\,\omega_0 \pm j0.25\,\omega_0)$

Figure 7.6.1 Illustration of the location of the poles of the Zero Input Mode ($\zeta_o = 0.866$), the Rate Control Mode (m = 1), and the Position Control Mode. The poles are shown for gimbal angles of zero and 30 degrees to show effects of change in gimbal angle. The open loop poles of the position control mode are the closed loop poles of the zero input mode.

CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

8.1 Summary of the General Concepts of the Thesis

The thesis is concerned with the problem of attitude control of a spacecraft engaged in an extended mission. The foremost application of such a spacecraft is a manned exploration of the planet Mars. The factors considered in choosing a suitable attitude control system are

Maximum reliability,

Minimum ejection of mass,

Minimum average power,

Minimum system weight, and

Minimum peak power.

Attitude control systems which do not expend fuel mass but derive control moments by a time rate of change in angular momentum of a mass that remains within the spacecraft are called momentum exchange type attitude control systems and consume only power in their operation. In a space environment, power is more available and may actually be re-supplied from the sun, whereas expulsion control fuel is limited to that initially loaded aboard the spacecraft. Based on this, the main theme of the thesis is the study of momentum exchange type attitude control systems.

The thesis proceeds to accomplish five objectives set forth in section 1.2. In short, these objectives are

derive the equations of motion,

determine the torque disturbances,

select a specific control system,

determine the response of the spacecraft, and present the conclusions.

The attitude control system chosen to best satisfy the spacecraft requirements is a configuration of four controllers which operate to apply gyroscopic torques to the spacecraft. Each controller may be described as a gyroscopic type device having two degrees-of-freedom. The four controllers are arranged in two pairs with each pair operating back-to-back. One pair of the controllers is actuated symmetrically to generate torques applied to roll the vehicle, whereas, the other pair, mounted normal to the first pair, provides torques for pitch control. All four of the controllers provide torques for yaw control. This configuration of four controllers has a redundancy in its ability to effect a change in angular momentum along each of the three spacecraft control axes such that the system provides uninterrupted control upon the complete loss of angular momentum of any one controller.

The four controller attitude control system is operated in three modes:

Zero Input Mode

Rate Control Mode

Position Control Mode.

The zero input mode enables the spacecraft to be rate stabilized in the absence of input commands. The rate control mode provides a means of changing the attitude of the vehicle at the maximum rate capability of the controllers. The position control mode is provided for alignment of the spacecraft with respect to a reference line of sight. Satisfactory stability characteristics were obtained for each mode.

8.2 Summary of Chapter Conclusions

Each chapter contains a brief summary, the highlights of which follow.

Chapter 1: This chapter introduces the problem of attitude control. In the practical problem the control system engineer must have knowledge of the physical characteristics of the spacecraft, the controllability requirements, and the stability requirements.

Chapter 2: Chapter 2 contains the equations of motion of the spacecraft. The torques generated by the control system depend on the attitude of the controller rotor with respect to the spacecraft and the rates and accelerations of the controller rotor with respect to inertial space. The equations are presented in a manner which facilitates the evaluation of each of these variables.

Chapter 3: One or more controllers are combined to form various attitude control systems. Control logic is required to give non-interacting control for the roll, pitch, and yaw vehicle attitude variables. A system of compensation is devised to minimize cross-coupling.

Chapter 4: This chapter seeks to determine the torque disturbances acting on a spacecraft. It is determined that the external torque disturbances can be minimized by design and operation of the spacecraft. The torques which originate within the spacecraft from masses which do not leave the system have a zero mean value and their effects can be compensated for by using a momentum exchange type attitude control system.

Chapter 5: This chapter compared the various systems and determined that the four gyro controller best fulfilled the control system requirements. The six gyro control system is the second choice. Inertia reaction wheel systems have very low power efficiency, and they lack the inherent stabilizing characteristics of the gyro controllers. Mass expulsion systems must be provided to some extent to desaturate the momentum exchange system, but if used continuously, large amounts of fuel will be required unless a limit cycle of the order of 15 minutes is provided.

Chapter 6: The control loop was closed using the four gyro controller. A general solution of the equations requires machine computation because of the nonlinearity and complexity of the equations. Three modes of operation are considered. Zero Input Mode, Rate Control Mode, and Position Control Mode.

Chapter 7: The response of the spacecraft is found to be satisfactory for all modes of operation using a fixed value of gimbal damping. The zero input mode and the rate control mode become less oscillatory at larger gimbal angles, whereas the position control mode becomes more oscillatory. The error signal from the sensor must be limited to avoid a possible unstable condition which drives the gimbal to their full 90 degree position.

8.3 Summary of Characteristics of Gyro Type Controllers

The following conclusions are presented concerning gyro type controllers used to provide attitude control of spacecraft.

a. The use of gyro controllers operating back-to-back eliminates large cross control moments typical of single gyro controllers, and this becomes more important as the gimbal angle is increased.

b. Gyro controllers should be operated without an appreciable amount of saturation in order to minimize gyroscopic cross coupling moments. This statement is true about any momentum exchange system, and suggests that zero angular momentum type systems are preferred to those which do not have zero angular momentum in their initial configuration.

c. Except for the unstable, full 90 degree position of the gyro controllers where no control moment is required, gyro controllers require continuous control moments to provide an attitude rate to the spacecraft.

d. During the angular acceleration of the spacecraft to an attitude rate the torque multiplication for the gyro torquer is high. The ideal torque gain for a pair of controllers operating open loop is equal to

$$\frac{1}{2\zeta_o}\sqrt{\frac{I_x}{J}}$$

e. Gyro controllers provide inherent rate stabilization.

f. Since the gyro gimbal is free to change its attitude with respect to the spacecraft, generally a control system using several gyro controllers will have inherent adaptive characteristics, in that, the failure of a single gyro unit will be compensated for by the repositioning of one or more of the other gyros.

g. In the absence of external torques the spacecraft assumes a particular attitude rate corresponding to a controlled gimbal position as given by the following equation for roll rate.

$$p = p_{max} \sin \gamma_1$$

h. Gyro controllers are more efficient than wheel controllers since they can effect a change in angular momentum of the spacecraft without a change in their kinetic energy, whereas wheel controllers require a kinetic energy change to effect an angular momentum change which results in a wheel efficiency ratio of the order of the ratio of the wheel moment of inertia to the spacecraft moment of inertia.

i. The provision of a position control loop for the gimbal angle, as is done in the rate control mode, causes a deterioration in the ability of the gyro to provide inherent rate stabilization.

j. The ratio of moment of inertia to damping coefficient for the gyro controller should be greater than that typically used for integrating gyros of inertial navigation systems.

k. The power requirement for a gyro controller is approximately constant as compared to an inertia wheel which has severe peak power requirements.

l. Gyro controllers can provide fast, accurate, and well damped control for manned spacecraft.

8.4 Recommendations for Further Study

As an extension to the thesis it is considered desirable to continue the study in the following areas.

a. Optimization of the design of gyro controllers.

b. Study of controllers which employ fluids to obtain a change in angular momentum.

c. By the use of machine computation, determine the effects of the cross coupling moments acting on the spacecraft.

d. Determine the feasibility of using existing angular moment such as power turbines in a spacecraft to achieve attitude control.

e. Investigate devices which can store energy as well as deliver energy for use with inertia reaction wheel control systems.

APPENDIX A

SYMBOLS AND MATRIX NOTATION

A.1 Matrix Notation

The treatment of an analytical problem that uses several sets of orthogonal Cartesian coordinates is simplified if matrix methods are employed so that the coordinate frames are unambiguously inferred by the matrix expressions. In reference 1 there is contained a particularly good explanation of matrix notation as applied to control system problems. Thus for detailed explanation the reader is referred to that reference and only a brief discussion of the notation is presented herein.

A.2 Coordinate Transformation Matrices

The symbol Q is defined as an orthogonal transformation between two Cartesian coordinate frames, and it is a 3 × 3 square matrix. Each Q shall contain a double subscript indicating the pertinent coordinate reference frames in a "to-from" sequence from left to right. For example, Q_{IA} is the coordinate transformation which is post multiplied by a vector in the A frame to transform the vector to the I frame. Q_{AI} then is the coordinate transformation that transforms a vector from the I frame to the A frame, and since all Q transformations are orthogonal transformations the operation of inverting a Q matrix is simply that of taking the transpose of the matrix.

$$Q_{AI} = \left[Q_{IA}\right]^{-1} = \left[Q_{IA}\right]^{T} \qquad \text{(Eq. A. 2. 1)}$$

Choice of the "to-from" sequence in subscripts facilitates multiple transformations so that the following is true.

$$Q_{IR} = Q_{IV}\, Q_{VA}\, Q_{AR} \qquad \text{(Eq. A. 2. 2)}$$

It is sometimes necessary to represent a coordinate frame by more than one letter, as for example the gimbal frame is referred to as GIM. When there is chance of ambiguity in a subscript a comma is used to separate the two frames. Thus the coordinate transformation $Q_{A,\,GIM}$ represents the coordinate transformation to the A frame from the GIM frame. The coordinate frames used in this thesis are contained in Appendix C.

A. 3 Relative Velocities

A velocity is a vector quantity and is expressed in matrix form by a column vector. Thus W_{IE} means the 3 × 1 matrix representing the angular velocity of frame E relative to frame I, expressed in frame E.

In matrix equations it is necessary to have a means of expressing the operation of a vector cross product. Thus consider a vector

$$\overline{W}_{IR} = l\bar{i} + m\bar{j} + n\bar{k} \qquad \text{(Eq. A. 3.1)}$$

and a vector

$$\overline{H}_R = A\bar{i} + B\bar{j} + C\bar{k} \qquad \text{(Eq. A. 32)}$$

The cross product
(Refer to ref 29 p 190)

$$\overline{W}_{IR} \times \overline{H}_R = \begin{bmatrix} \bar{i} & \bar{j} & \bar{k} \\ l & m & n \\ A & B & C \end{bmatrix} \qquad \text{(Eq. A. 33)}$$

$$\overline{W}_{IR} \times \overline{H}_R = (mC - nB)\,\bar{i} + (nA - lC)\,\bar{j} + (lB - mA)\,\bar{k} \qquad \text{(Eq. A. 34)}$$

In matrix form the result of the cross product operation can be represented as a column matrix such that,

$$\left. W_{IR} \times H_R \right]_R = \begin{bmatrix} mC - nB \\ nA - lC \\ lB - mA \end{bmatrix} \qquad \text{(Eq. A. 3.5)}$$

Then upon separation of the variables associated with W_{IR} and $\left.H_R\right]_R$ respectively gives:

$$W_{IR}^{\star} \left.H_R\right]_R = \begin{bmatrix} 0 & -n & m \\ n & 0 & -1 \\ -m & 1 & 0 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} \qquad \text{(Eq. A. 3.6)}$$

Therefore we define from any column matrix such as

$$W_{IR} = \begin{bmatrix} 1 \\ m \\ n \end{bmatrix} \qquad \text{(Eq. A. 3.7)}$$

the 3 × 3 antisymmetric matrix

$$W_{IR}^{\star} \equiv \begin{bmatrix} 0 & -n & m \\ n & 0 & -1 \\ -m & 1 & 0 \end{bmatrix} \qquad \text{(Eq. A. 3.8)}$$

Reference 1 derives this definition in a more rigorous manner from the fundamental operation of differentiating a matrix equation. Suppose

$$W_{IR} = Q_{RA} W_{IA} + W_{AR} \qquad \text{(Eq. A. 3.9)}$$

Differentiating with respect to time

$$\dot{W}_{IR} = Q_{RA} \dot{W}_{IA} + \dot{Q}_{RA} W_{IA} + \dot{W}_{AR} \qquad \text{(Eq. A. 3.10)}$$

Since $Q_{RA} Q_{AR} = I$, a matrix Q_{RA} can be factored from equation A. 310.

$$\dot{W}_{IR} = Q_{RA} \left[\dot{W}_{IA} + Q_{AR} \dot{Q}_{RA} W_{IA} \right] + \dot{W}_{AR} \qquad \text{(Eq. A. 3.11)}$$

and by differentiating each of the elements of Q_{RA} and premultiplying the result by Q_{AR} we find that

$$Q_{AR}\,\dot{Q}_{RA} \text{ is a } 3 \times 3 \text{ antisymmetric matrix}$$

of the form

$$Q_{AR}\,\dot{Q}_{RA} = W_{RA}^{\star} \text{ as defined in equation A. 3.8.}$$

A. 4 Numbering of Controller Elements

When control systems are considered that contain many elements some system must be devised to keep track of the various elements. For the lack of a better arrangement the controllers have simply been numbered as the problems have been solved. Therefore, the following is a list of the various numbers assigned to the controllers (Gyro).

1	Gyro with Spin Reference Axis along	y axis
2	Gyro with Spin Reference Axis along	-y axis
3	Gyro with Spin Reference Axis along	x axis
4	Gyro with Spin Reference Axis along	-x axis
5	Gyro with Spin Reference Axis along	z axis
6	Gyro with Spin Reference Axis along	-z axis
7	Gyro with Spin Reference Axis in x-y plane and rotated 120° from Gyro 1 about z axis.	
8	Gyro with Spin Reference Axis in x-y plane and rotated -120° from Gyro 1 about z axis.	
9	Gyro with Spin Reference Axis in y-z plane and rotated $-\alpha_o$ degrees from -y axis about x axis	
10	Gyro with Spin Reference Axis in y-z plane and rotated $+\alpha_o$ degrees from -y axis about x axis	

A particular control system may require single-degree-of-freedom controllers, two-degree-of-freedom controllers, or possibly three-degree-of-freedom controllers in which the angular speed of the rotor may be varied in addition to the two degrees-of-freedom in precession of the spin axis of the controller[33]. Therefore the controllers used for a particular system require further specification to fully define their configuration. Accordingly in Appendix G which presents specific control systems the angles required to align the case of the controller shown in Figure B. 9 of Appendix B are given for each system. See also Appendix C. 3 for coordinate transformations for particular controllers.

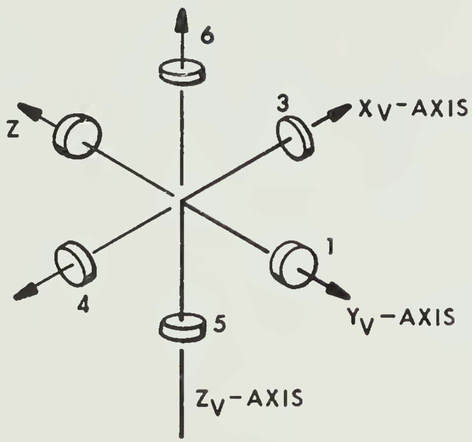


Figure A. 41 Illustrating general positions of Controllers 1 through 6.

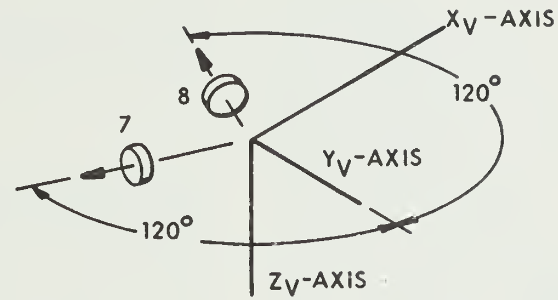


Figure A. 42 illustrating general positions of Controllers 7 and 8.

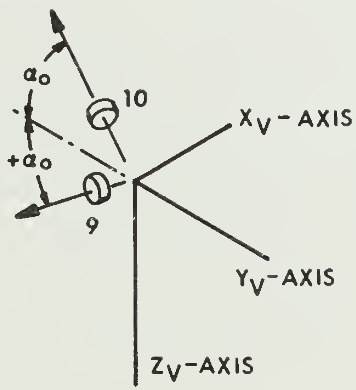


Figure A. 43 illustrating general positions of Controllers 9 and 10.

A. 5 List of Symbols

α Angle used to Define Gimbal Position in Case. See Figure B. 10.

γ Angle used to Define Case Position Relative to Vehicle. See Figure B. 9. Also used as a General Control System Variable.

β Angle used to Define Rotor Position Relative to Gimbal. See Figure B. 11.

η_x, η_y, & η_z Represent Primary Control Variables. Also used as angle without subscript in Figure B. 3.

♈ Represents Vernal Equinox.

Ω, ω, & i Angular Rotations of Heliocentric Orbital Plane Reference Frame relative to Heliocentric Inertial Reference Frame.

ϕ, θ and ψ Angular Rotations of vehicle-centered Vehicle Reference frame with respect to an Inertial Reference Frame.

λ_K, μ_K, & μ_{KP} Refers to Angular Rotations of Geocentric Orbital Position Reference Frame relative to the Geocentric Orbital Plane Reference Frame. See Figure B. 13.

LAT Latitude. See Figure B. 14.

LON Longitude. See Figure B. 14

μ_s and Λ_s Angular Rotations used in Defining the Geocentric Solar Reference Frame. See Figure B. 15.

A Represents an Angle when provided with a subscript. When used as a subscript the symbol refers to the Vehicle Principal Axis Reference Frame. See Figure B. 8.

E Energy

H Angular Momentum

M Moment

P Power

Q Orthogonal Coordinate Transformation Matrix. (See Appendix C)

R Resistance, ohms

p, q, and r Vehicle Attitude Rates in Roll, Pitch, and Yaw

W_{IA} Angular Velocity of Coordinate Frame A relative to Coordinate Frame I expressed as a Column Matrix

I Moment of Inertia of Spacecraft

J Moment of Inertia of Components of Control System

n Refers to number of columns or rows of a matrix

● A dot over a symbol indicates a differentiation with respect to time

☆ Represents the forming of a Matrix from a Column Vector. See Section A. 3.

S Shorthand notation for the trigonometric sine.

C Shorthand notation for the trigonometric cosine.

Subscripts

T Total

CS Control System

ext External

VF Refers to Masses in the Vehicle that are Rigidly Fixed to the Spacecraft

VM Refers to Masses in the Vehicle that Move Relative to the Vehicle Exclusive of the Rotating Member of the Control System Controller.

N Represents a summation symbol for N Controllers

a Armature

i Summation Index. Also used in defining the Heliocentric Orbital Plane Reference Frame. See Figure B. 2.

c Refers to the Case of the Controller

g Refers to the Gimbal of the Controller

r Refers to the Rotor of the Controller

x, y, and z Generally refers to the x, y, and z direction of an Orthogonal Coordinate Reference Frame.

w Wheel

Zϕ, YN, and XV Refers to Angular Rotations of Vehicle-Centered Vehicle Reference Frame relative to Vehicle-Centered Inertial Reference Frame. See Figure B. 7.

ZV, 1; YY; and XA Refers to Angular Rotations of Vehicle-Centered Principal Axis Frame relative to Vehicle-Centered Vehicle Reference Frame. See Figure B. 8.

YU and X, GU Refers to Angular Rotations of Vehicle-Centered Gyro Case Axis Reference Frame relative to Vehicle-Centered Vehicle Reference Frame. See Figure B. 9.

Z1 Refers to angular rotation of Earth. See Figure B. 12.

See section B. 1 for Symbols used for Coordinate Reference Frames.

A. 5 Glossary of Terms

Active Control

Active control of a spacecraft is defined as the operation of controlling the spacecraft with a torque producing control system operating with error sensors in a closed loop. The control system always consumes energy.

Compensation

A signal provided to a control loop which is proportional to a signal of another loop, and is used to minimize the effects of an unwanted coupling between these two loops. Three types of compensation are used in this report: gyroscopic coupling, cross control coupling, and spacecraft inertia cross coupling.

Control Logic Matrix

The control logic matrix is defined as an $n \times 3$ matrix which pre-multiplies the primary control variables to define individual signals to the n-degree-of-freedom controller.

Control System Coupling Matrix

The control system coupling matrix is a 3×3 matrix which operates on the vehicle rate variables resulting from the combination of one or more controllers. If the control system coupling matrix contains no diagonal terms then it is also the gyroscopic coupling matrix.

Control System Input Matrix

A $3 \times n$ matrix which results from any arrangement of terms of one or more controllers in which the control system input matrix operates on the control system input variables.

Control System Input Variables

Any controller may have one or more degrees of freedom which may be considered the input variables of the controller, i. e. with wheels, the input variables are angular acceleration of the wheels. With pure gyro systems the input variables are precession rates. A combination of two or more controllers give many degrees of freedom which represent the control system input variables.

Controller

A momentum exchange device which is capable of applying control moments to the spacecraft. A controller in this report is considered to be a rotating rigid body. One or more controllers are combined to form a complete control system.

Gyroscopic Coupling Matrix

Defined as the control system coupling matrix with all diagonal elements replaced by zeros.

Guidance

The guidance of a vehicle is defined as the operation of controlling the thrust vectors acting on a spacecraft, such that, a desired trajectory is followed. Guidance of a spacecraft is required during thrusting and possibly during re-entry if the aerodynamic lift of the vehicle can be controlled.

Inertial Guidance Measurement Unit

The inertial guidance measurement unit is defined as an assemblage of instrumentation to determine the specific force vectors acting on the spacecraft suitable for performing the guidance and navigation function.

Momentum Exchange

A momentum exchange control system is defined as a device which is capable of providing control torques to a spacecraft by a time rate of change in angular momentum within the device. Consequently, the total mass and angular momentum of the combined vehicle and control system remains constant.

Momentum Transfer

A momentum transfer control system is defined as a device which is capable of providing control torques to a spacecraft by ejecting mass from the spacecraft and creating a force-impulse normal to a lever arm directed to the center of gravity of the spacecraft.

Navigation

Navigation of a spacecraft is defined as the operation of determination of the position and velocity (or position and velocity deviations), and computation of the guidance commands necessary to arrive at the desired destination.

Rest Point

A spacecraft rest point is the attitude which results in zero applied torque, and at which attitude the spacecraft exhibits stable static stability characteristics.

Passive Control

Passive control of a spacecraft is defined as the operation of controlling the spacecraft purely by means of an existing stabilizing torque which acts on the spacecraft. Attitude control is achieved by dissipating energy and the system provides inherent sensing.

Pointing Accuracy

The pointing accuracy is defined as the maximum deviation

of the output of a closed loop with reference to a desired output when no disturbances are admitted to the loop. Pointing accuracy is a measure of the stability of a spacecraft controlled by an active control system.

Primary Control Matrix

The primary control matrix is a result of pre-multiplying the control logic matrix by the control system input matrix. The primary control matrix is then a 3×3 matrix and is diagonal or nearly diagonal for the range of control system input variables chosen to define the control logic matrix.

Primary Control Variables

Three variables which give non-interacting (or nearly non-interacting) control in roll, pitch, and yaw.

Rest Position

For a spacecraft in an environment in any time invariant situation, there exists a spacial orientation at which the external moments acting on the spacecraft exactly cancel and no moment acts on the vehicle.

Semi-Passive Control

Semi-passive control of a spacecraft is defined as passive control systems which increase their damping properties by introducing gyroscopic effects, or those systems which possess no static stability but have strong damping characteristics such as spin stabilized vehicles.

Saturate (and desaturate)

Any momentum exchange type control system will have an initial angular momentum disposition and a maximum angular momentum storage capability for a particular control axis. Saturation is defined as the percent change in the angular momentum of a particular axis when the

vehicle is non-rotating with respect to the reference frame. Desaturation is defined as providing an external moment to the spacecraft which tends to return the momentum exchange control system to its initial disposition.

Vehicle Attitude Rate Variables

The vehicle attitude rate variables are the roll, pitch, and yaw rates of the vehicle with respect to inertial space.

APPENDIX B

COORDINATE REFERENCE FRAMES

B. 1 Summary of Coordinate Reference Frames Defined

Figure	Symbol	Title
B. 1	II	Heliocentric Inertial Reference Frame
B. 2	H	Heliocentric Orbital Plane Reference Frame
B. 3	B	Heliocentric Orbital Position Reference Frames
B. 4	ϕ, θ, ψ	Vehicle-Centered Solar Orbital Reference Frame
B. 5	I	Vehicle-Centered Inertial Reference Frame
B. 6	VR	Vehicle-Centered Velocity Reference Frame
B. 7	V	Vehicle-Centered Vehicle Reference Frame
B. 8	A	Vehicle-Centered Principal Axis Frame
B. 9	GU	Vehicle-Centered Gyro Case Axis Reference Frame
B. 10	GIM	Vehicle-Centered Gyro Gimbal Axis Reference Frame
B. 11	R	Vehicle-Centered Gyro Rotor Axis Reference Frame
B. 12	III	Geocentric Inertial Non-rotating Reference Frame, and
	E	Geocentric Earth Reference Frame
B. 13	K P	Geocentric Orbital Plane Reference Frame, and Geocentric Orbital Position Reference Frame

B. 14	G	Geocentric Longitude-Latitude Grid Reference Frame
B. 15	S	Geocentric Solar Reference Frame
B. 16	O	Vehicle-Centered Planet Orbital Reference Frame

An attempt has been made to keep the defined reference frames identical with those given by Ogletree in reference 21. For reference frames centered at a planet of the solar system other than earth, it is considered that Figures B. 12 and B. 16 can be applied with a suitable subscript denoting the planet concerned.

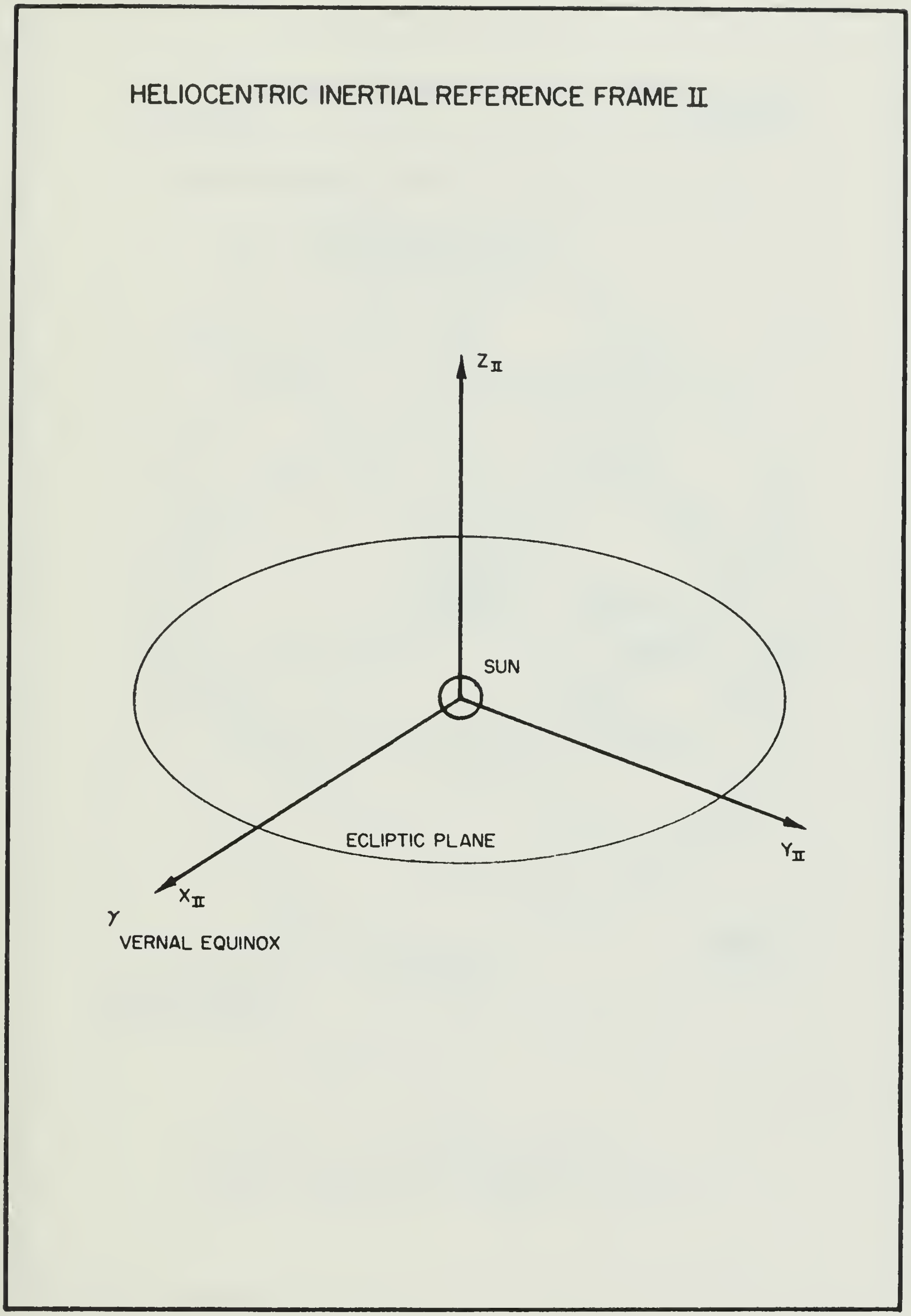
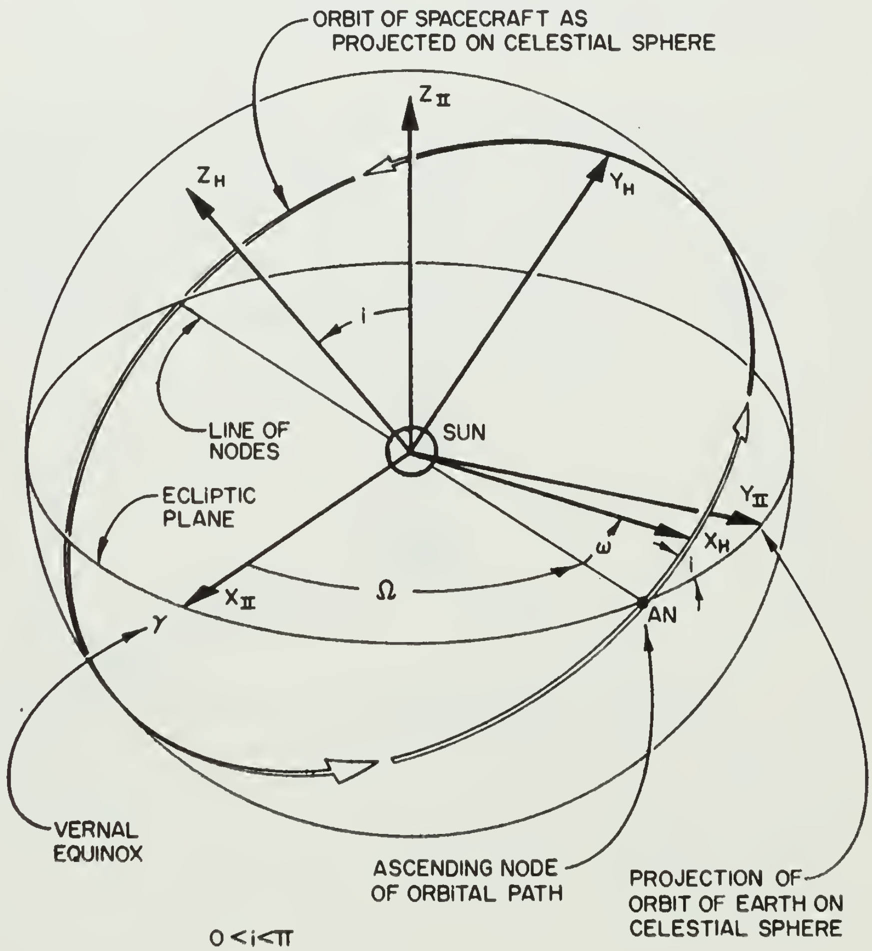


FIGURE B.1 An illustration of an inertial fixed frame assuming the sun as a fixed point in space

HELIOCENTRIC ORBITAL PLANE REFERENCE FRAME H

AXIS X_H PASSES THROUGH PERIHELION OF SPACECRAFT ORBIT

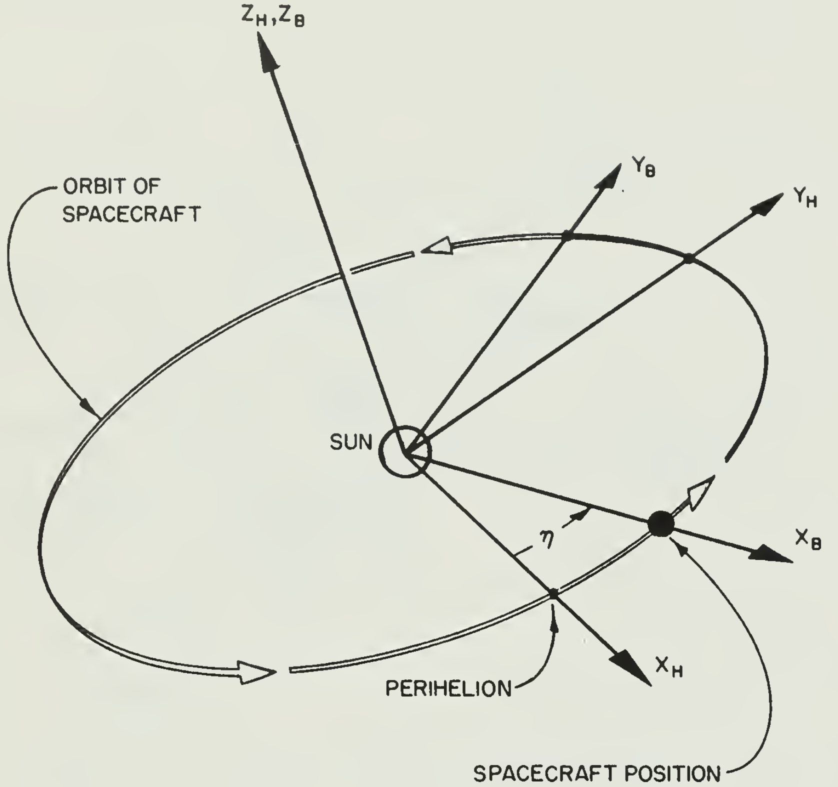


$0 < i < \pi$

$$Q_{H,\mathrm{II}} = \begin{bmatrix} (C\omega C\Omega - S\omega S\Omega Ci) & (C\omega S\Omega + S\omega C\Omega Ci) & (S\omega Si) \\ (-S\omega C\Omega - C\omega S\Omega Ci) & (-S\omega S\Omega + C\omega C\Omega Ci) & (SiC\omega) \\ SiS\Omega & -SiC\Omega & Ci \end{bmatrix}$$

FIGURE B.2 An Illustration of an Orbital Plane Reference Frame defined as Three Rotations from the Heliocentric Inertial Reference Frame.

HELIOCENTRIC ORBITAL POSITION REFERENCE FRAME B

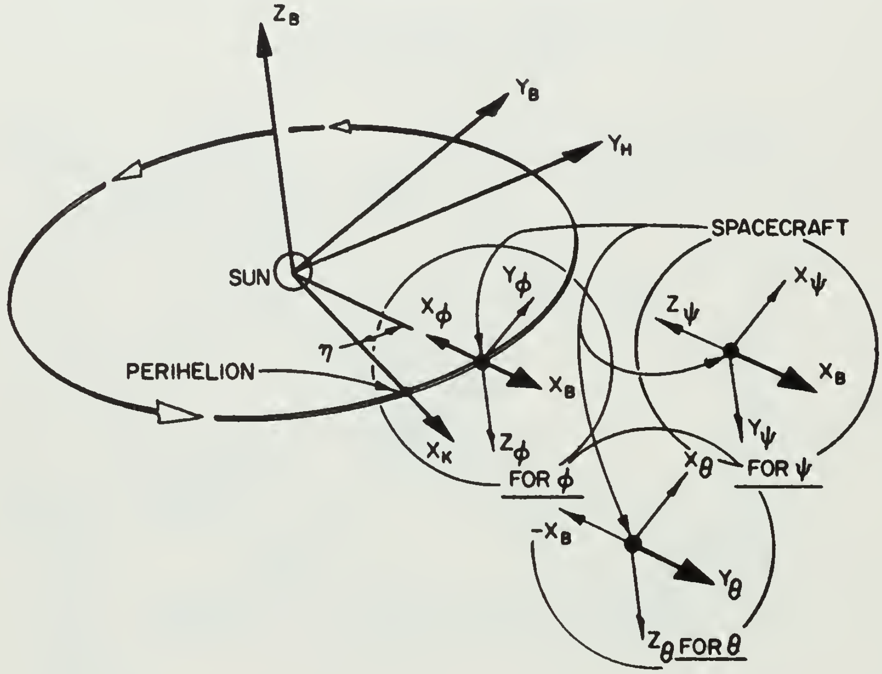


$$W_{HB} = \begin{bmatrix} 0 \\ 0 \\ \dot{\eta} \end{bmatrix}$$

$$Q_{BH} = \begin{bmatrix} +\mathrm{Cos}\,\eta & +\mathrm{Sin}\,\eta & 0 \\ -\mathrm{Sin}\,\eta & +\mathrm{Cos}\,\eta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

FIGURE B.3 An illustration of a Heliocentric Orbital Position Reference Frame defined by a Single Rotation from the Heliocentric Orbital Plane Reference Frame.

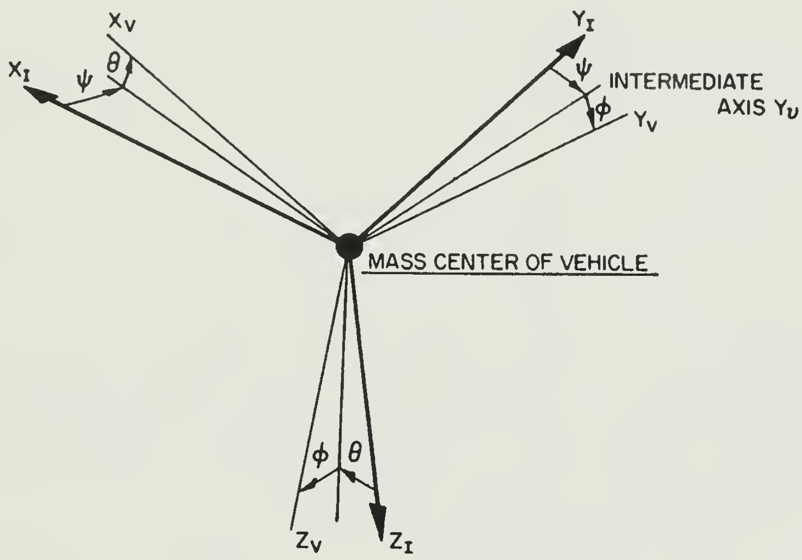
VEHICLE-CENTERED SOLAR ORBITAL REFERENCE FRAME (THREE CASES) ϕ, θ, & ψ



$$Q_{\phi B} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad Q_{\theta B} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad Q_{\psi B} = \begin{bmatrix} 0 & +1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

FIGURE B.4 An Illustration of a Vehicle-Centered Orbital Reference Frame as related to the Heliocentric Orbital Position Reference Frame.

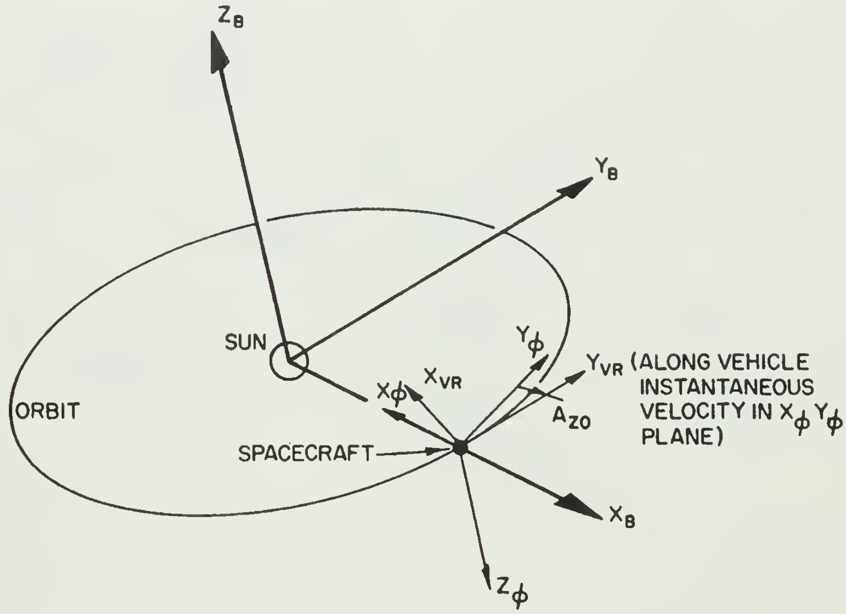
VEHICLE-CENTERED VEHICLE REFERENCE FRAME V SHOWN RELATIVE TO VEHICLE-CENTERED INERTIAL REFERENCE FRAME I



1. FRAME I IS NON-ROTATING WITH RESPECT TO FIXED STARS, I.E. WITH RESPECT TO FRAME II.
2. DIRECTIONS OF X_I, Y_I, Z_I ARE DEFINED TO COINCIDE WITH X_0, Y_0, Z_0 AT TIME ZERO FOR ANY PARTICULAR SET OF INITIAL CONDITIONS.
3. ORDER OF ROTATIONS TO PLACE FRAME I IN COINCIDENCE WITH FRAME V.
   - A. ROTATE ABOUT AXIS Z_I THROUGH ANGLE ψ.
   - B. ROTATE ABOUT AXIS Y_υ THROUGH ANGLE θ.
   - C. ROTATE ABOUT AXIS X_V THROUGH ANGLE ϕ.

FIGURE B-5 An illustration of relation between the Vehicle-Centered Inertial Reference Frame and the Vehicle-Centered Vehicle Reference Frame.

VEHICLE-CENTERED VELOCITY-REFERENCE FRAME VR

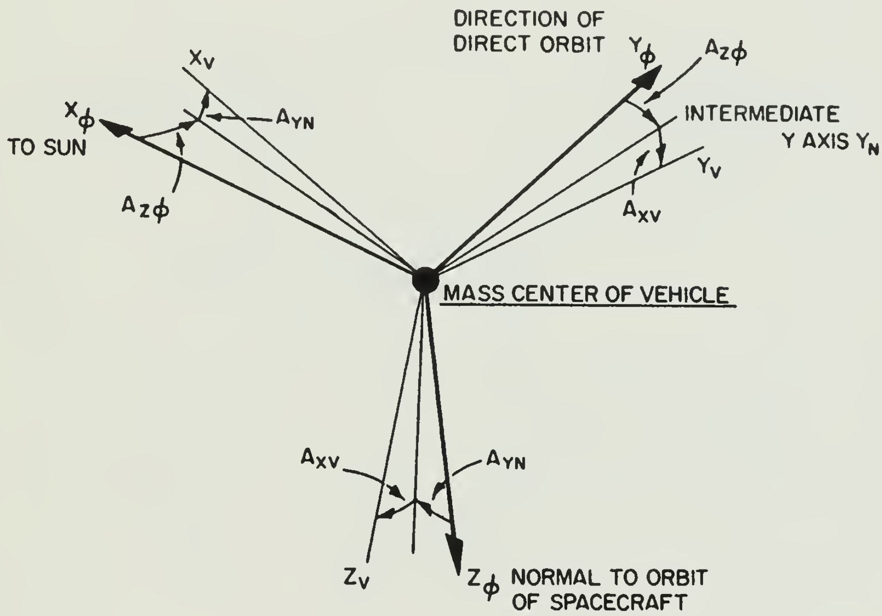


$$W_{VR,\phi} = \begin{bmatrix} 0 \\ 0 \\ \dot{A}_{ZO} \end{bmatrix}$$

$$Q_{VR,\phi} = \begin{bmatrix} +CA_{ZO} & +SA_{ZO} & 0 \\ -SA_{ZO} & +CA_{ZO} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

FIGURE B.6 An illustration of a Vehicle-Centered Velocity-Reference Frame. Angle A_{ZO} is rotation about Z to place Frame I in coincidence with Frame VR.

VEHICLE-CENTERED VEHICLE REFERENCE FRAME V



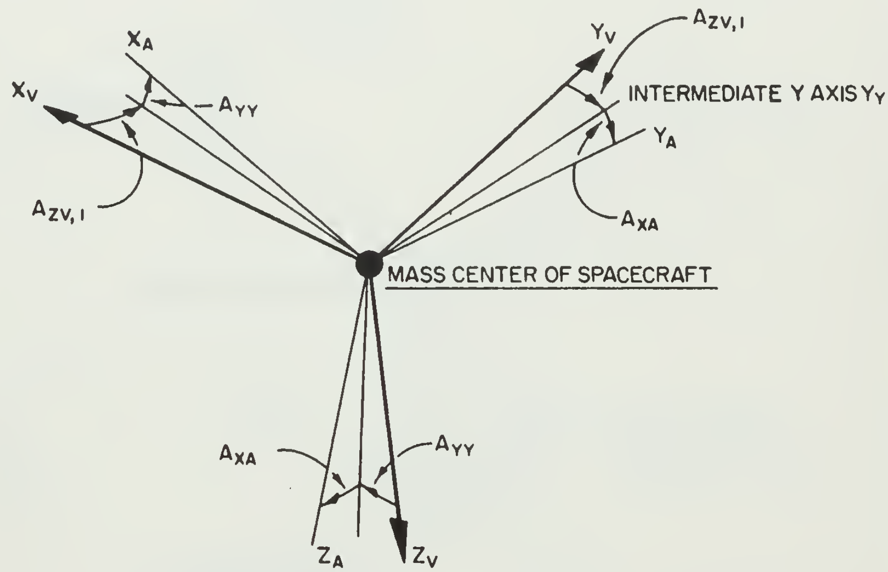
ORDER OF ROTATIONS TO PLACE FRAME ϕ IN COINCIDENCE WITH FRAME V.

1. ROTATE ABOUT AXIS Z_ϕ THROUGH ANGLE $A_{Z\phi}$.
2. ROTATE ABOUT AXIS Y_N THROUGH ANGLE A_{YN}.
3. ROTATE ABOUT AXIS X_V THROUGH ANGLE A_{XV}.

SEE APPENDIX C FOR $Q_{V\phi}$.

FIGURE B.7 An illustration of a Vehicle-Centered Vehicle Reference Frame as defined by three rotations from the Vehicle-Centered Orbital Reference Frame.

VEHICLE-CENTERED PRINCIPAL AXIS FRAME A



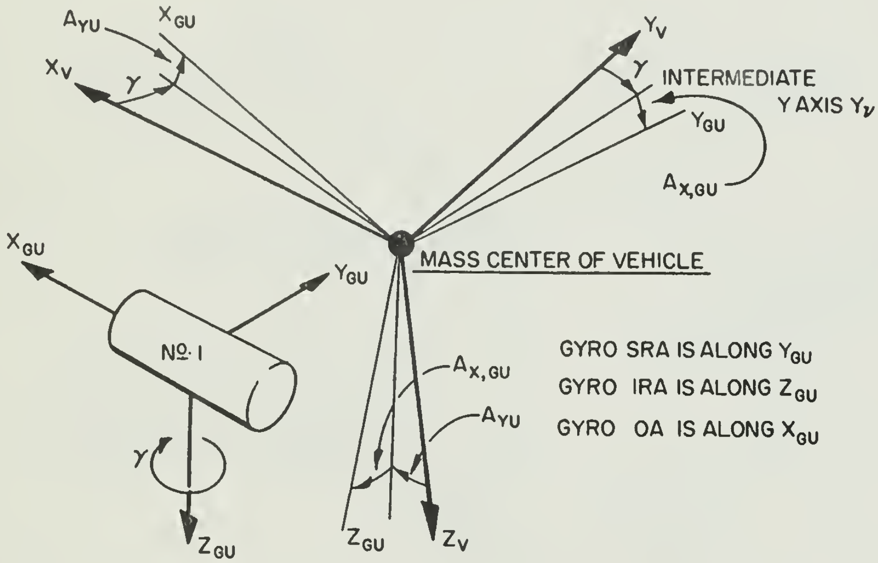
ORDER OF ROTATIONS TO PLACE FRAME V IN COINCIDENCE WITH FRAME A.

1. ROTATE ABOUT AXIS Z_V THROUGH ANGLE $A_{ZV,1}$.
2. ROTATE ABOUT AXIS Y_Y THROUGH ANGLE A_{YY}.
3. ROTATE ABOUT AXIS X_A THROUGH ANGLE A_{XA}.

SEE APPENDIX C FOR Q_{AV}.

FIGURE B.8 An Illustration of a Vehicle-Centered Principal Axis Frame as defined by three rotations from the Vehicle-Centered Vehicle Reference Frame.

VEHICLE-CENTERED GYRO CASE AXIS REFERENCE FRAME GU



GYRO SRA IS ALONG Y_{GU}

GYRO IRA IS ALONG Z_{GU}

GYRO OA IS ALONG X_{GU}

ORDER OF ROTATIONS TO PLACE FRAME V IN COINCIDENCE WITH FRAME GU.

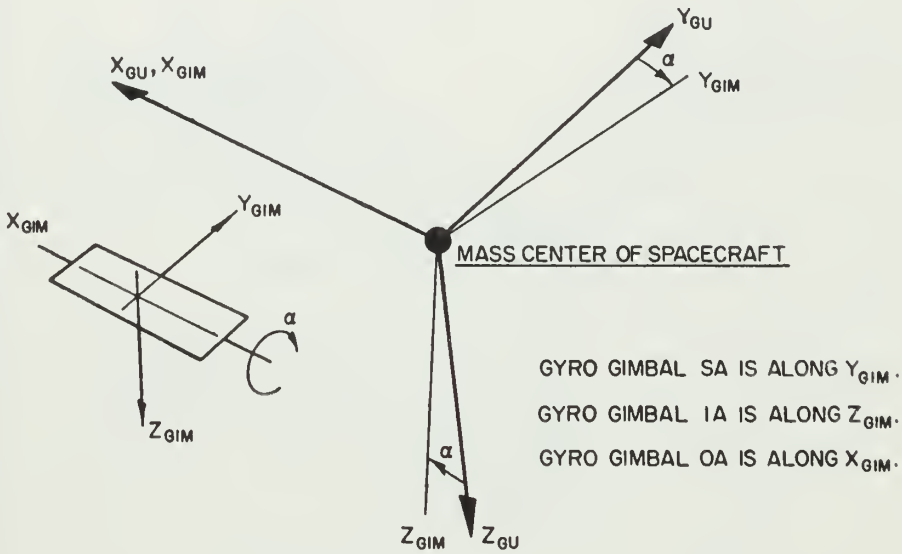
1. ROTATE ABOUT AXIS Z_V THROUGH ANGLE γ.
2. ROTATE ABOUT AXIS Y_ν THROUGH ANGLE A_{YU}.
3. ROTATE ABOUT XIS X_{GU} THROUGH ANGLE $A_{X,GU}$.

   SEE APPENDIX C FOR $Q_{GU,V}$.

$$\overline{W}_{V,GU} \cong \begin{bmatrix} 0 \\ 0 \\ \dot{\gamma} \end{bmatrix} \qquad J_{C_{GU}} = \begin{bmatrix} J_{CX} & 0 & 0 \\ 0 & J_{CY} & 0 \\ 0 & 0 & J_{CZ} \end{bmatrix}$$

FIGURE B.9 An illustration of a Vehicle-Centered Gyro Case Axis Reference Frame as defined by three rotations from the Vehicle-Centered Vehicle Reference Frame.

VEHICLE-CENTERED GYRO GIMBAL AXIS REFERENCE FRAME GIM



TO PLACE FRAME GU IN COINCIDENCE WITH FRAME GIM.

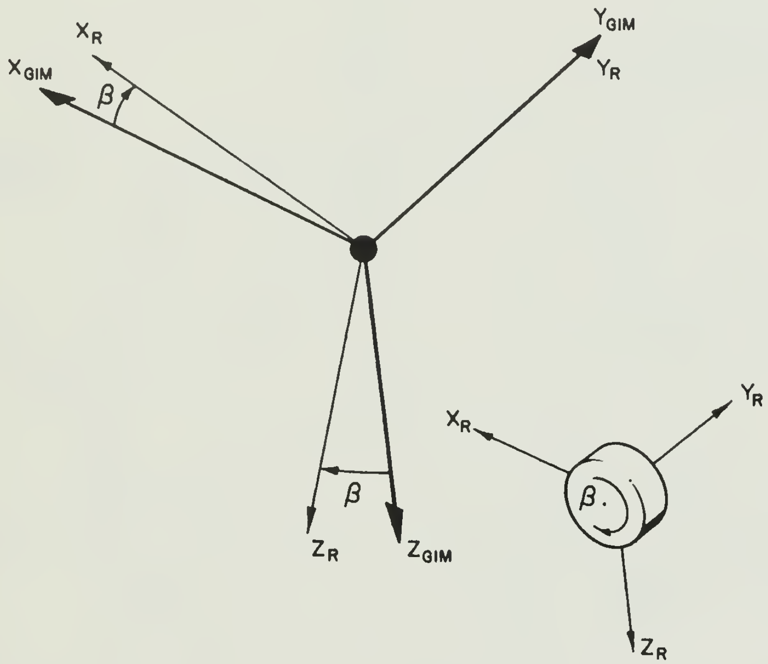
1. ROTATE ABOUT AXIS X_{GU} THROUGH ANGLE α.

$$\overline{W}_{GU,GIM} = \begin{bmatrix} \dot{\alpha} \\ 0 \\ 0 \end{bmatrix} \qquad J_{g,GIM} = \begin{bmatrix} J_{gX} & 0 & 0 \\ 0 & J_{gY} & 0 \\ 0 & 0 & J_{gZ} \end{bmatrix}$$

$$Q_{GIM,GU} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & +\cos\alpha & +\sin\alpha \\ 0 & -\sin\alpha & +\cos\alpha \end{bmatrix}$$

FIGURE B.10 An illustration of a Vehicle-Centered Gyro Gimbal Axis Reference Frame as defined by a single rotation from the Vehicle-Centered Gyro Case Axis Reference Frame.

VEHICLE-CENTERED GYRO ROTOR AXIS REFERENCE FRAME R



TO PLACE FRAME GIM IN COINCIDENCE WITH FRAME R

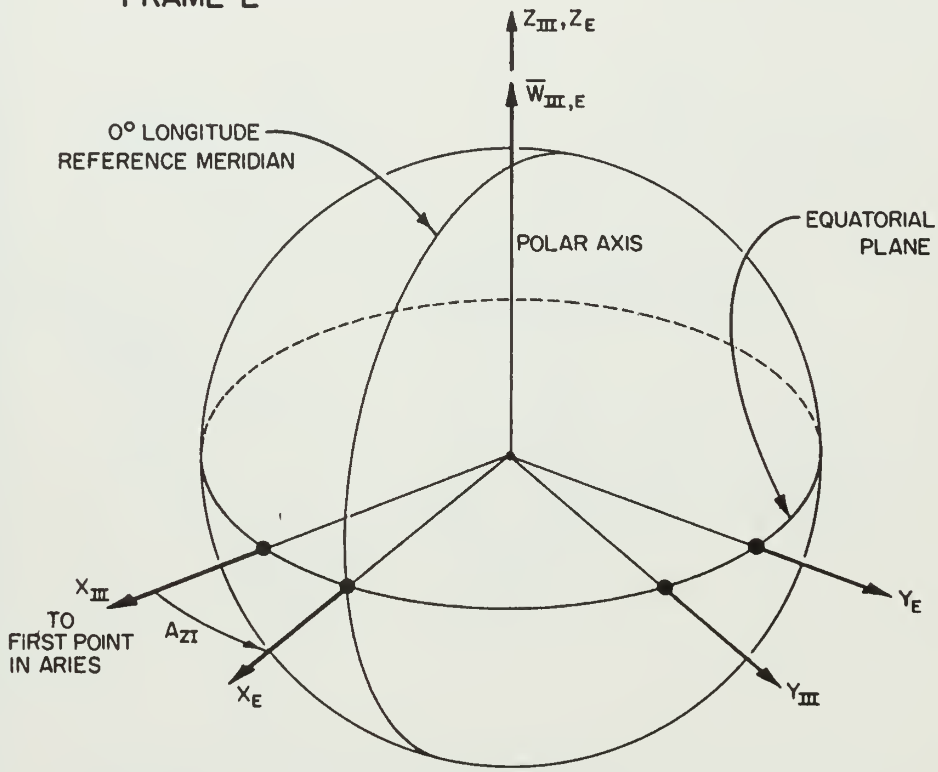
1. ROTATE ABOUT AXIS Y_{GIM} THROUGH ANGLE β.

$$\overline{W}_{GIM,R} = \begin{bmatrix} 0 \\ \dot{\beta} \\ 0 \end{bmatrix} \qquad J_{rR} = \begin{bmatrix} J_{rX} & 0 & 0 \\ 0 & J_{rY} & 0 \\ 0 & 0 & J_{rZ} \end{bmatrix}$$

$$Q_{R,GIM} = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ +\sin\beta & 0 & \cos\beta \end{bmatrix}$$

FIGURE B.11 An illustration of a Vehicle-Centered Gyro Rotor Axis Reference Frame as defined by a single rotation from the Vehicle-Centered Gyro Gimbal Axis Reference Frame.

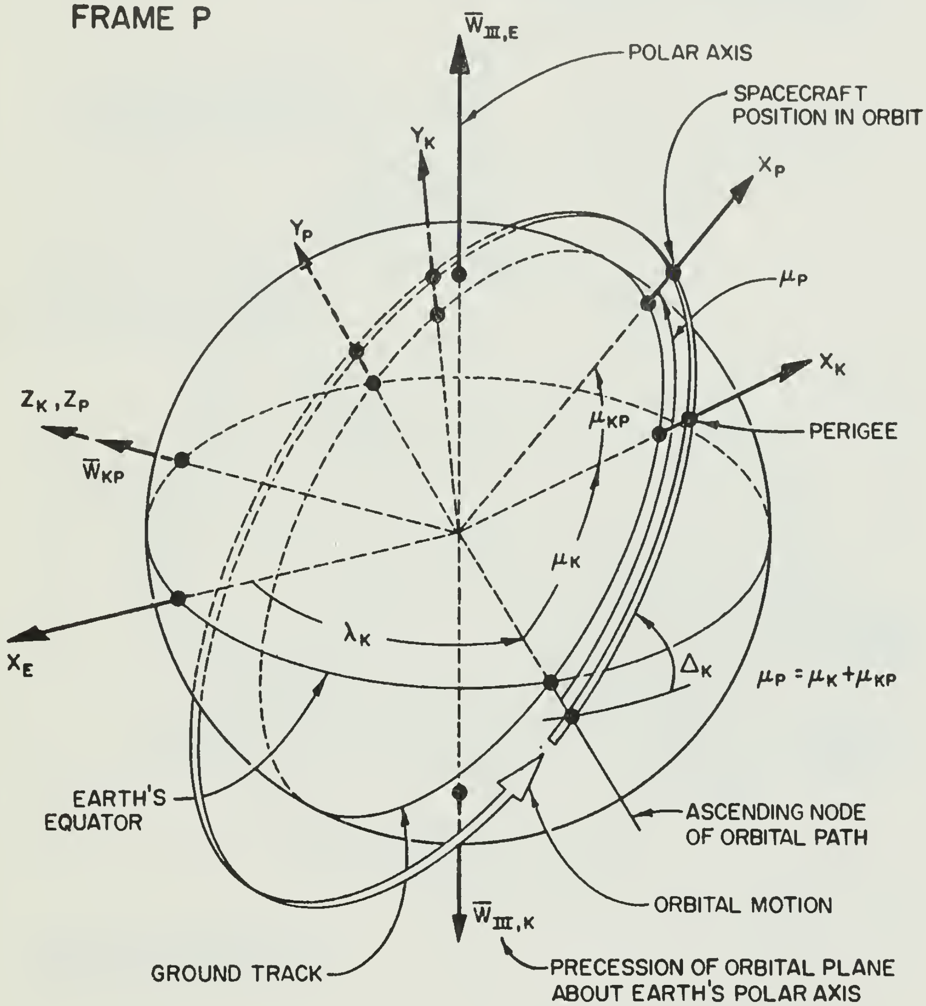
GEOCENTRIC INERTIAL NON-ROTATING REFERENCE FRAME III AND GEOCENTRIC EARTH REFERENCE FRAME E



$$Q_{E,III} = \begin{bmatrix} CA_{ZI} & SA_{ZI} & 0 \\ -SA_{ZI} & CA_{ZI} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad W_{III,E} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{2\pi \text{ RAD}}{86{,}164 \text{ SEC}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 7.292115 \times 10^{-5} \dfrac{\text{RAD}}{\text{SEC}} \end{bmatrix}$$

FIGURE B-12 An illustration of the relations between a Geocentric Inertial Non-Rotating Reference Frame and a Geocentric Earth Reference Frame which rotates at Earth Rate.

GEOCENTRIC ORBITAL PLANE REFERENCE FRAME K AND GEOCENTRIC ORBITAL POSITION REFERENCE FRAME P



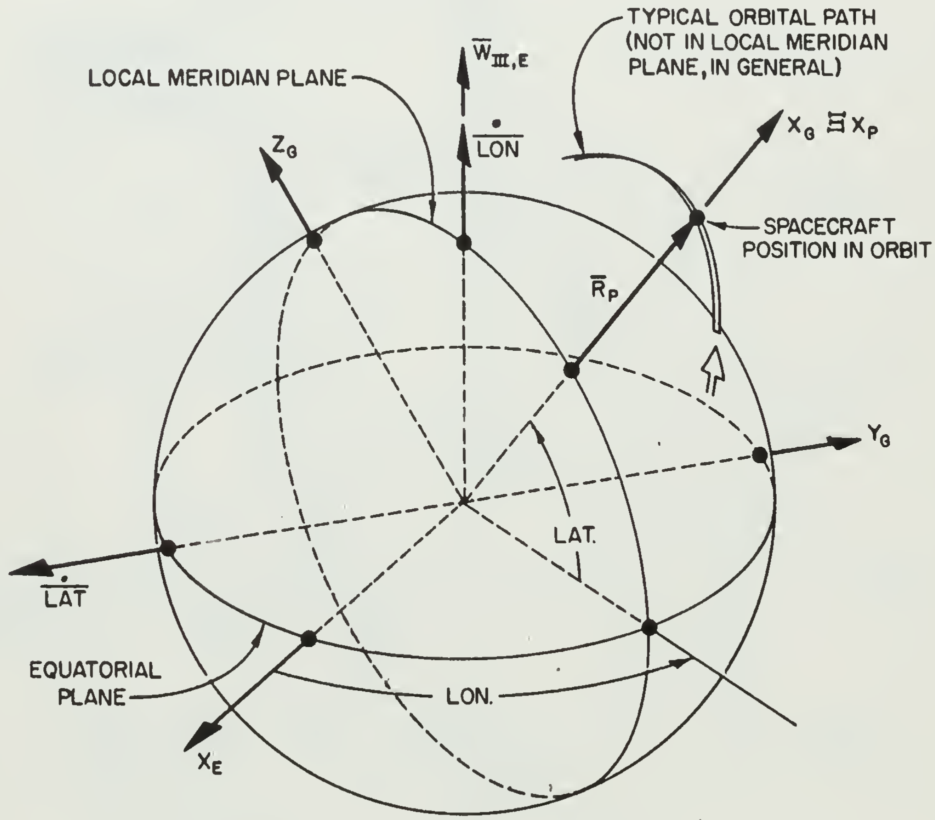
$\lambda_K = -(W_{IK} + W_{IE})(T - T_0) + \lambda_K\rfloor_0$

T = INITIAL VALUE OF TIME

$\lambda_K\rfloor_0$ = INITIAL LONGITUDE OF ASCENDING NODE OF ORBIT

FIGURE B.13 An illustration of the relations between a Geocentric Orbital Plane and a Geocentric Orbital Position Reference Frame

GEOCENTRIC LONGITUDE-LATITUDE GRID
REFERENCE FRAME G



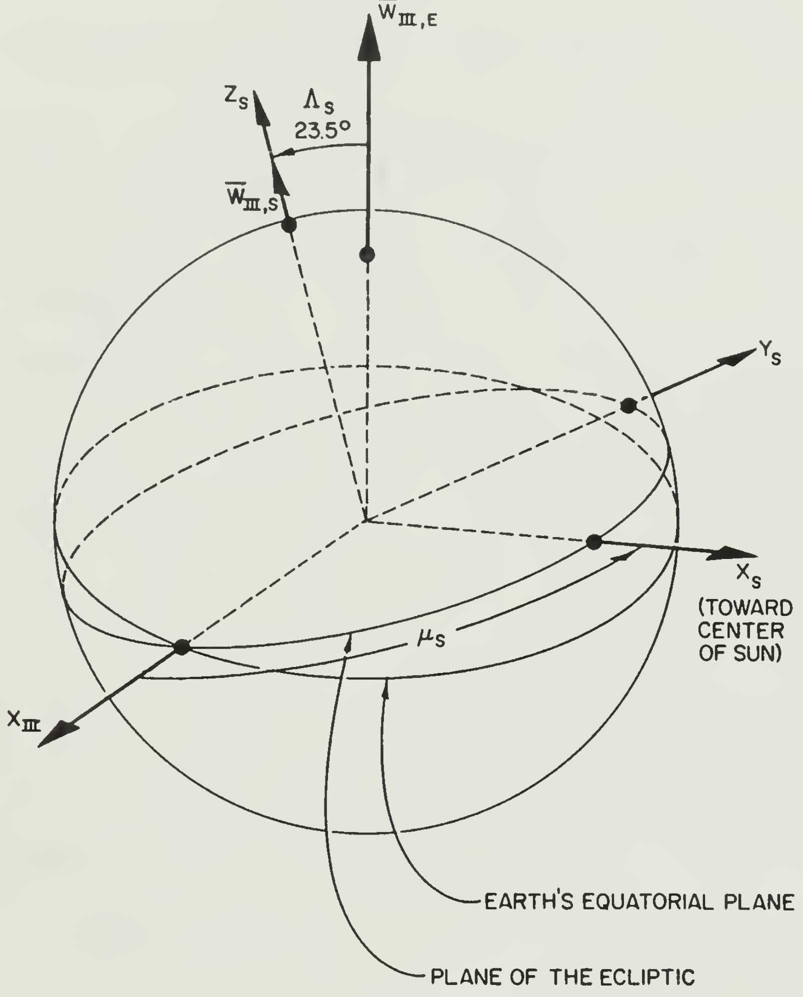
$\overline{R}_P$ IS GEOCENTRIC RADIUS VECTOR TO SPACECRAFT

Y IS IN DIRECTION OF $\overline{W}_{IE} * \overline{R}_P$

$$P_{GE} = \begin{bmatrix} C_{LAT}C_{LON} & C_{LAT}S_{LON} & S_{LAT} \\ -S_{LON} & C_{LON} & 0 \\ -S_{LAT}C_{LON} & -S_{LAT}C_{LON} & C_{LAT} \end{bmatrix}$$

FIGURE B.14 An Illustration of a Geocentric Longitude-Latitude Grid Reference Frame.

GEOCENTRIC SOLAR REFERENCE FRAME S



$\Lambda_S = 23°26'59''$

$\overline{W}_{III,S} = \frac{2\pi \text{ RAD}}{365.25 \text{ DAYS}}$

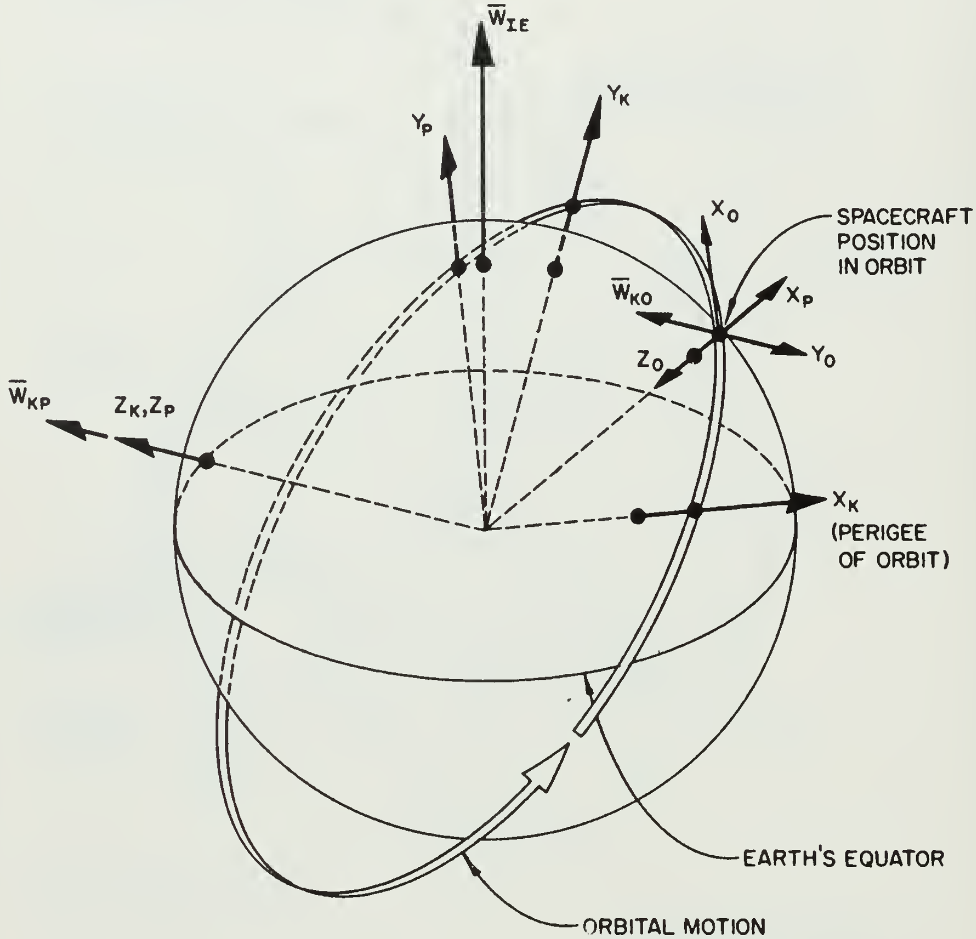
$$Q_{S,III} = \begin{bmatrix} C\mu_S & S\mu_S C\Lambda_S & S\mu_S S\Lambda_S \\ -S\mu_S & C\mu_S C\Lambda_S & C\mu_S S\Lambda_S \\ 0 & -S\Lambda_S & C\Lambda_S \end{bmatrix}$$

$\mu_S = \mu_S\rfloor_0 + W_{IS}(T - T_0)$

$\mu_S\rfloor_0$ = INITIAL VALUE OF μ_S

FIGURE B.15 An Illustration of a Geocentric Solar Reference Frame.

VEHICLE-CENTERED PLANET ORBITAL REFERENCE FRAME O



$\overline{W}_{KO} \equiv \overline{W}_{KP}$

$\hat{X}_O \equiv \hat{Y}_P$

$\hat{Y}_O \equiv \hat{Z}_P$

$\hat{Z}_O \equiv \hat{X}_P$

$$Q_{OP} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

FIGURE B.16 An illustration of a Vehicle-Centered Orbital Reference Frame.

APPENDIX C

COORDINATE TRANSFORMATIONS

C. 1 Coordinate Transformations Between Reference Frames Relating to Interplanetary Space Analysis

(Refer to Appendix B for definitions of coordinate frames)

$$Q_{H,II} = \begin{bmatrix} (C\omega C\Omega - S\omega S\Omega C i) & (C\omega S\Omega + S\omega C\Omega C i) & S\omega S i \\ (-S\omega C\Omega - C\omega S\Omega C i) & (-S\omega S\Omega + C\omega C\Omega C i) & S i C\omega \\ S i S\Omega & -S i C\Omega & C i \end{bmatrix}$$

(Eq. C. 1. 1)

$$Q_{BH} = \begin{bmatrix} Cn & Sn & 0 \\ -Sn & Cn & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{(Eq. C. 1. 2)}$$

$$Q_{\phi B} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

See Figure B. 4 for $Q_{\theta B}$ and $Q_{\psi B}$

(Eq. C. 1. 3)

$$Q_{VR,\phi} = \begin{bmatrix} CA_{ZO} & SA_{ZO} & 0 \\ -SA_{ZO} & CA_{ZO} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{(Eq. C. 1. 4)}$$

$$
Q_{V\phi} = \begin{bmatrix} CA_{YN}\,CA_{ZO} & CA_{YN}\,SA_{ZO} & -SA_{YN} \\ \left(-CA_{XV}\,SA_{ZO} + SA_{XV}\,SA_{YN}\,CA_{ZO}\right) & \left(CA_{XV}\,CA_{ZO} + SA_{XV}\,SA_{YN}\,SA_{ZO}\right) & SA_{XV}\,CA_{YN} \\ \left(SA_{XV}\,SA_{ZO} + CA_{XV}\,SA_{YN}\,CA_{ZO}\right) & \left(-SA_{XV}\,CA_{ZO} + CA_{XV}\,SA_{YN}\,SA_{ZO}\right) & CA_{XV}\,CA_{YN} \end{bmatrix} \qquad \text{(Eq. C. 15)}
$$

$$
Q_{AV} = \begin{bmatrix} CA_{YY} CA_{ZV,1} & CA_{YY} SA_{ZV,1} & -SA_{YY} \\ -CA_{XA} SA_{ZV,1} + SA_{X,A} SA_{YY} CA_{ZV,1} & CA_{XA} CA_{ZV,1} + SA_{XA} SA_{YY} SA_{ZV,1} & SA_{XA} CA_{YY} \\ SA_{XA} SA_{ZV,1} + CA_{XA} SA_{YY} CA_{ZV,1} & -SA_{XA} CA_{ZV,1} + CA_{XA} SA_{YY} SA_{ZV,1} & CA_{XA} CA_{YY} \end{bmatrix}
$$

(Eq. C. 16)

$$Q_{GU,V} = \begin{bmatrix} CA_{YU}C\gamma & CA_{YU}S\gamma & -SA_{YU} \\ \left(-CA_{X,GU}S\gamma + SA_{X,GU}SA_{YU}C\gamma\right) & \left(CA_{X,GU}C\gamma + SA_{X,GU}SA_{YU}S\gamma\right) & SA_{X,GU}CA_{YU} \\ \left(SA_{X,GU}S\gamma + CA_{X,GU}SA_{YU}C\gamma\right) & \left(-SA_{X,GU}C\gamma + CA_{X,GU}SA_{YU}S\gamma\right) & CA_{X,GU}CA_{YU} \end{bmatrix}$$

(Eq. C. 1.7)

$$Q_{GIM,\,GU} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & +C\alpha & +S\alpha \\ 0 & -S\alpha & +C\alpha \end{bmatrix} \qquad \text{(Eq. C. 1. 8)}$$

$$Q_{R,\,GIM} = \begin{bmatrix} +C\beta & 0 & -S\beta \\ 0 & 1 & 0 \\ +S\beta & 0 & C\beta \end{bmatrix} \qquad \text{(Eq. C. 1. 9)}$$

$$Q_{R,\,GU} = \begin{bmatrix} C\beta & S\alpha\, S\beta & -C\alpha\, S\beta \\ 0 & C\alpha & S\alpha \\ S\beta & -S\alpha\, C\beta & C\alpha\, C\beta \end{bmatrix} \qquad \text{(Eq. C. 1. 10)}$$

$$Q_{VI} = \begin{bmatrix} C\theta\, C\psi & C\theta\, S\psi & -S\theta \\ -C\phi S\psi + S\phi S\theta C\psi & C\phi S\psi + S\phi S\theta S\psi & S\phi\, C\theta \\ S\phi S\psi + C\phi S\theta C\psi & -S\phi C\psi + C\phi S\theta S\psi & C\phi\, C\theta \end{bmatrix} \qquad \text{(Eq. C. 1. 11)}$$

TRANSFORMATIONS FOR SPECIAL CASE

$A_{YU} = 0 \qquad A_{X,GU} = 0 \qquad Q_{AV} = I$

$$Q_{GU,A} = \begin{bmatrix} C\gamma & S\gamma & 0 \\ -S\gamma & C\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{(Eq. C. 1. 12)}$$

$$Q_{GIM,A} = \begin{bmatrix} C\gamma & S\gamma & 0 \\ -S\gamma C\alpha & C\alpha C\gamma & S\alpha \\ S\gamma S\alpha & -S\alpha C\gamma & C\alpha \end{bmatrix} \qquad \text{(Eq. C. 1. 13)}$$

$$Q_{RA} = \begin{bmatrix} C\beta C\gamma - S\alpha S\beta S\gamma & C\beta S\gamma + S\alpha S\beta C\gamma & -C\alpha S\beta \\ -C\alpha S\gamma & C\alpha C\gamma & S\alpha \\ S\beta C\gamma + S\alpha C\beta S\gamma & S\beta S\gamma - S\alpha C\beta C\gamma & C\alpha C\beta \end{bmatrix}$$

(Eq. C. 1. 14)

C. 2 Coordinate Transformations Between Reference Frames Relating to Planetary Space Analysis

$$Q_{E, III} = \begin{bmatrix} CA_{Z1} & SA_{Z1} & 0 \\ -SA_{Z1} & CA_{Z1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{(Eq. C. 2. 1)}$$

$$Q_{KE} = \begin{bmatrix} \left(C\mu_K C\lambda_K - S\mu_K C\Lambda_K S\lambda_K\right) & \left(C\mu_K S\lambda_K + S\mu_K C\Lambda_K C\lambda_K\right) & S\mu_K S\Lambda_K \\ \left(-S\mu_K C\lambda_K - C\mu_K C\Lambda_K S\lambda_K\right) & \left(-S\mu_K S\lambda_K + C\mu_K C\Lambda_K C\lambda_K\right) & C\mu_K S\Lambda_K \\ \left(S\Lambda_K S\lambda_K\right) & \left(-S\Lambda_K C\lambda_K\right) & C\Lambda_K \end{bmatrix} \qquad \text{(Eq. C. 2. 2)}$$

$$Q_{PK} = \begin{bmatrix} C\mu_{KP} & S\mu_{KP} & 0 \\ -S\mu_{KP} & C\mu_{KP} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{(Eq. C. 2. 3)}$$

$$
Q_{GE} = \begin{bmatrix} \text{C LAT C LON} & \text{C LAT S LON} & \text{S LAT} \\ \text{-S LON} & \text{C LON} & 0 \\ \text{-S LAT C LON} & \text{-S LAT S LON} & \text{C LAT} \end{bmatrix}
$$

(Eq. C. 2. 4)

$$
Q_{S,\,III} = \begin{bmatrix} C\mu_s & S\mu_s C\Lambda_s & S\mu_s S\Lambda_s \\ -S\mu_s & C\mu_s C\Lambda_s & C\mu_s S\Lambda_s \\ 0 & -S\Lambda_s & C\Lambda_s \end{bmatrix}
$$

(Eq. C. 2. 5)

$$
Q_{OP} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}
$$

(Eq. C. 2. 6)

$$
Q_{OE} = \begin{bmatrix} \left(-S\mu_P C\lambda_K - C\mu_P C\Lambda_K S\lambda_K\right) & \left(-S\mu_P S\lambda_K + C\mu_P C\Lambda_K C\lambda_K\right) & C\mu_P S\Lambda_K \\ -S\Lambda_K S\lambda_K & S\Lambda_K C\lambda_K & -C\Lambda_K \\ \left(-C\mu_P C\lambda_K + S\mu_P C\Lambda_K S\lambda_K\right) & \left(-C\mu_P S\lambda_K - S\mu_P C\Lambda_K C\lambda_K\right) & -S\mu_P S\Lambda_K \end{bmatrix}
$$

(Eq. C. 2. 7)

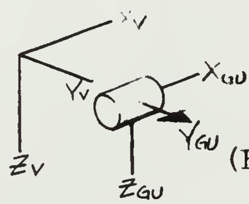
$$Q_{VO} = \begin{bmatrix} CA_{YN}CA_{ZO} & CA_{YN}SA_{ZO} & -SA_{YN} \\ \left(-CA_{XV}SA_{ZO} + SA_{XV}SA_{YN}CA_{ZO}\right) & \left(CA_{XV}CA_{ZO} + SA_{XV}SA_{YN}SA_{ZO}\right) & SA_{XV}CA_{YN} \\ \left(SA_{XV}SA_{ZO} + CA_{XV}SA_{YN}CA_{ZO}\right) & \left(-SA_{XV}CA_{ZO} + CA_{XV}SA_{YN}SA_{ZO}\right) & CA_{XV}CA_{YN} \end{bmatrix} \qquad \text{(Eq. C. 2. 8)}$$

NOTE: $Q_{VO} \equiv Q_{V\phi}$

C. 3 Coordinate Transformations Relating to the Definition of Particular Controllers

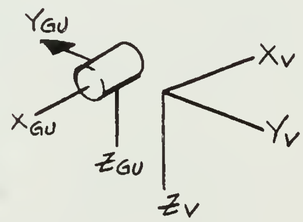
TWO-DEGREE-OF-FREEDOM CONTROLLERS

$$Q_{GU\,1,\,V} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



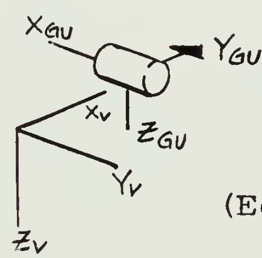
(Eq. C. 3. 1)

$$Q_{GU\,2,\,V} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



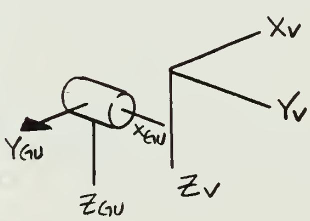
(Eq. C. 3. 2)

$$Q_{GU\,3,\,V} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



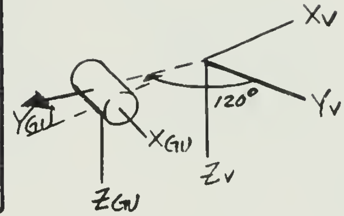
(Eq. C. 3. 3)

$$Q_{GU\,4,\,V} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



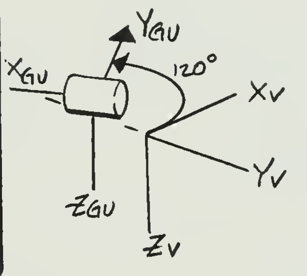
(Eq. C. 3. 4)

$$Q_{GU\,7,\,V} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



(Eq. C. 3. 5)

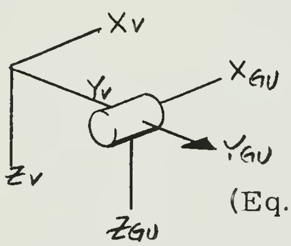
$$Q_{GU\,8,\,V} = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



(Eq. C. 3. 6)

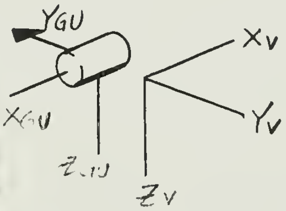
SINGLE-DEGREE-OF-FREEDOM CONTROLLERS

$$Q_{GU\,1,\,V} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



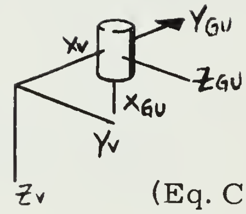
(Eq. C. 3. 7)

$$Q_{GU\,2,\,V} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



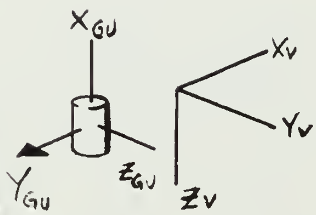
(Eq. C. 3. 8)

$$Q_{GU\,3,\,V} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



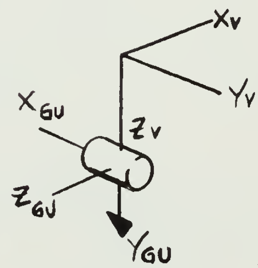
(Eq. C. 3. 9)

$$Q_{GU\,4,\,V} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



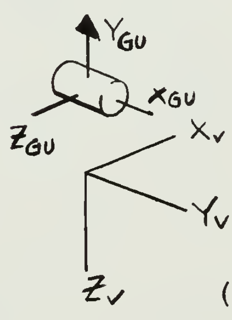
(Eq. C. 3. 10)

$$Q_{GU\,5,\,V} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & +1 \\ -1 & 0 & 0 \end{bmatrix}$$



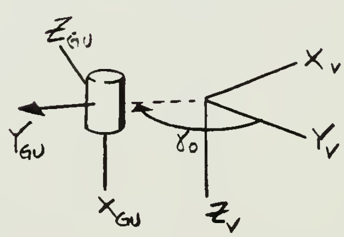
(Eq. C. 3. 11)

$$Q_{GU\,6,\,V} = \begin{bmatrix} 0 & +1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$



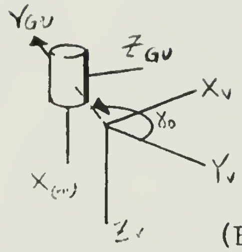
(Eq. C. 3. 12)

$$Q_{GU\,7,\,V} = \begin{bmatrix} 0 & 0 & +1 \\ -S\gamma_o & +C\gamma_o & 0 \\ -C\gamma_o & -S\gamma_o & 0 \end{bmatrix}$$



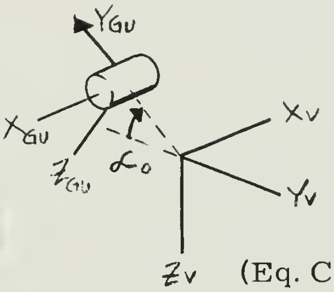
(Eq. C. 3. 13)

$$Q_{GU\,8,\,V} = \begin{bmatrix} 0 & 0 & +1 \\ +S\gamma_o & +C\gamma_o & 0 \\ -C\gamma_o & +S\gamma_o & 0 \end{bmatrix}$$



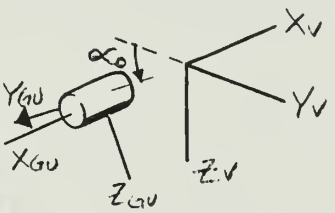
(Eq. C. 3. 14)

$$Q_{GU\,9,\,V} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -C\alpha_o & -S\alpha_o \\ 0 & -S\alpha_o & +C\alpha_o \end{bmatrix}$$



(Eq. C. 3. 15)

$$Q_{GU\,10,\,V} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -C\alpha_o & +S\alpha_o \\ 0 & +S\alpha_o & +C\alpha_o \end{bmatrix}$$



(Eq. C. 3. 16)

APPENDIX D

D. 1 Relative Velocities

$W_{II,H} = 0$ Assuming Orbital Elements are Constant

(Eq. D. 1. 1)

$$W_{HB} = \begin{bmatrix} 0 \\ 0 \\ \dot{\eta} \end{bmatrix}$$

where $\dot{\eta} = \dfrac{r_\pi V_\pi}{r^2}$ of the order of 1 degree/day.

(Eq. D. 1. 2)

r_π = radius of perihelion of spacecraft orbit

V_π = velocity of spacecraft at perihelion

r = radius of spacecraft from sun

$$W_{B\phi} = 0$$

(Eq. D. 1. 3)

$$W_{\phi V} = \begin{bmatrix} 1 & 0 & -SA_{YN} \\ 0 & CA_{XV} & SA_{XV}CA_{YN} \\ 0 & -SA_{XV} & CA_{XV}CA_{YN} \end{bmatrix} \begin{bmatrix} \dot{A}_{XV} \\ \dot{A}_{YN} \\ \dot{A}_{ZO} \end{bmatrix}$$

(Eq. D. 1. 4)

$$W_{IV} = \begin{bmatrix} 1 & 0 & -S\theta \\ 0 & C\phi & S\phi C\theta \\ 0 & -S\phi & C\phi C\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

(Eq. D. 1. 5)

$$W_{V,GU} = \begin{bmatrix} 1 & 0 & -SA_{YN} \\ 0 & CA_{XGU} & SA_{XGU}\,CA_{YU} \\ 0 & -SA_{XGU} & CA_{XGU}\,CA_{YU} \end{bmatrix} \begin{bmatrix} \dot{A}_{X,GU} \\ \dot{A}_{YU} \\ \dot{\gamma} \end{bmatrix}$$

(Eq. D. 1. 6)

$$W_{GU,\,GIM} = \begin{bmatrix} \dot{\alpha} \\ 0 \\ 0 \end{bmatrix}$$

(Eq. D. 1. 7)

$$W_{GIM,\,R} = \begin{bmatrix} 0 \\ \dot{\beta} \\ 0 \end{bmatrix}$$

(Eq. D. 1. 8)

$$W_{VR,\,\phi} = \begin{bmatrix} 0 \\ 0 \\ \dot{A}_{ZO} \end{bmatrix}$$

(Eq. D. 1. 9)

$$W_{III,\,E} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{2\pi}{86,164} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 7.292115 \times 10^{-5} \text{ rad/sec} \end{bmatrix}$$

(Eq. D. 1. 10)

$W_{KE} = 0$ Assuming Orbital Elements are Constant

(Eq. D. 1. 11)

$$W_{PK} = \begin{bmatrix} 0 \\ 0 \\ \mu_{KP} \end{bmatrix} \qquad \text{(Eq. D. 1. 12)}$$

$$W_{OP} = 0 \qquad \text{(Eq. D. 1. 13)}$$

$$W_{III, S} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{2\pi \text{ Rad}}{365.25 \text{ days}} \end{bmatrix} \qquad \text{(Eq. D. 1. 14)}$$

D. 2 Approximate Relative Velocities

Valid when the frames nearly coincide or when angular velocity is purely about a single axis. The subscript A may be substituted for V for the case where $Q_{VA} = I$.

$$W_{\phi V} = \begin{bmatrix} \dot{A}_{XV} \\ \dot{A}_{YN} \\ \dot{A}_{ZO} \end{bmatrix} \qquad \text{(Eq. D. 2. 1)}$$

$$W_{IV} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \text{ which is further defined as a column matrix of vehicle attitude rate variables } \cong \begin{bmatrix} p \\ q \\ r \end{bmatrix} \qquad \text{(Eq. D. 2. 2)}$$

$$W_{V, GU} = \begin{bmatrix} \dot{A}_{X, GU} \\ \dot{A}_{YU} \\ \dot{\gamma} \end{bmatrix} \qquad \text{(Eq. D. 2. 3)}$$

$$W_{V, GU} = \begin{bmatrix} 0 \\ 0 \\ \dot{\gamma} \end{bmatrix} \text{ for the particular problem where the case has only one degree of freedom.} \qquad \text{(Eq. D. 2. 4)}$$

$$W_{V, GIM} = \begin{bmatrix} \dot{\alpha} \\ 0 \\ 0 \end{bmatrix} \text{ for the particular problem where } \dot{\alpha} \gg \dot{\gamma} \qquad \text{(Eq. D. 2. 5)}$$

$$W_{VR} = \begin{bmatrix} 0 \\ \dot{\beta} \\ 0 \end{bmatrix}$$ for the particular problem where $\beta \gg \dot{\alpha}$ and $\beta \gg \dot{\gamma}$ which assumes the rotor spin as large.

(Eq. D. 2. 6)

D.3 Relative Velocities for Controller 1

For the special case where the controller is in position 1, i.e. $A_{X, GU} = 0$, $A_{YU} = 0$, and γ is a control variable, the following relative velocities apply.

$$W_{V, GU} = \begin{bmatrix} 0 \\ 0 \\ \dot{\gamma} \end{bmatrix} \qquad \text{(Eq. D. 3. 1)}$$

$$W_{V, GIM} = \begin{bmatrix} \dot{\alpha} \\ \dot{\gamma} S \alpha \\ \dot{\gamma} C\alpha \end{bmatrix} \qquad \text{(Eq. D. 3. 2)}$$

$$W_{VR} = \begin{bmatrix} \dot{\alpha} C \beta - \dot{\gamma} C \alpha S \beta \\ \dot{\beta} + \dot{\gamma} S \alpha \\ \dot{\alpha} S \beta + \dot{\gamma} C \alpha C \beta \end{bmatrix} \qquad \text{(Eq. D. 3. 3)}$$

$$W_{IR} = \begin{bmatrix} p\left(C\beta C\gamma - S\alpha S\beta S\gamma\right) + q\left(C\beta S\gamma + S\alpha S\beta C\gamma\right) - r\left(C\alpha S\beta\right) - \dot{\gamma} C\alpha S\beta + \dot{\alpha} C\beta \\ -p\, C\alpha S\gamma + q\, C\alpha C\gamma + rS\alpha + \dot{\gamma} S\alpha + \dot{\beta} \\ p\left(S\beta C\gamma + S\alpha C\beta S\gamma\right) + q\left(S\beta S\gamma - S\alpha C\beta C\gamma\right) + rC\alpha C\beta + \dot{\gamma} C\alpha C\beta + \dot{\alpha} S\beta \end{bmatrix}$$

(Eq. D. 3. 4)

$$W_{I, GU} = \begin{bmatrix} p\, C\gamma + q\, S\gamma \\ -p\, S\gamma + q\, C\gamma \\ r + \gamma \end{bmatrix} \qquad \text{(Eq. D. 3. 5)}$$

$$
W_{I,\,GIM} = \begin{bmatrix} p\,C\gamma + q\,S\gamma + \dot{\alpha} \\ \left(-p\,S\gamma + q\,C\gamma\right) C\alpha + \left(r + \dot{\gamma}\right) S\alpha \\ \left(+p\,S\gamma - q\,C\gamma\right) S\alpha + \left(r + \dot{\gamma}\right) C\alpha \end{bmatrix} \qquad \text{(Eq. D.3.6)}
$$

D. 4 Time Rate of Change of Angular Velocities

For the case where $\dot{\beta} \gg \dot{\alpha}, \dot{\gamma}, \dot{p}, \dot{q}$ and $\dot{r}$. Controller 1

$$
W_{IR} = \begin{bmatrix} \{p(-S\beta C\gamma - S\alpha C\beta S\gamma) + q(-S\beta S\gamma + S\alpha C\beta C\gamma) - r\, C\alpha C\beta - \dot{\gamma} C\alpha C\beta - \dot{\alpha} S\beta\}\, \dot{\beta} \\ -p(C\alpha C\gamma \dot{\gamma} - S\alpha S\gamma \dot{\alpha}) - q(C\alpha S\gamma \dot{\gamma} + S\alpha C\gamma \dot{\alpha}) + r\, C\alpha \dot{\alpha} + \ddot{\gamma} S\alpha + \dot{\gamma} C\alpha \dot{\alpha} \\ \{p(C\beta C\gamma - S\alpha S\beta S\gamma) + q(C\beta S\gamma + S\alpha S\beta C\gamma) - r\, C\alpha S\beta - \dot{\gamma} C\alpha S\beta + \dot{\alpha} C\beta\}\, \dot{\beta} \end{bmatrix}
$$

(Eq. D. 4. 1)

also $\ddot{\beta} = 0$

APPENDIX E

EXACT MOMENT EQUATIONS FOR CASE, GIMBAL AND ROTOR TERMS

In accordance with the definition of the GU, the GIM, and the R coordinate reference frames of Appendix B, the case containing a gimbaled rotor is chosen so that the spin reference axis of the rotor is aligned along the positive y axis of the V frame. This gyro position is also called gyro 1. It is assumed that the V frame (vehicle-centered vehicle reference frame) and the A frame (vehicle-centered principle axis frame) coincide.

Equation E. 3. 1 is applicable to problems concerning inertia reaction wheels, but it is not particularly suited for wheels. Accordingly, section F. 2 of Appendix F is a derivation of an equation specifically for inertia reaction wheels.

APPENDIX E

EXACT EQUATIONS FOR CASE, GIMBAL, AND ROTOR TERMS

E. 1 Case Terms

From Equation 2. 3. 6

$$\sum M_{case\ 1}\rfloor_A = Q_{A,GU}\left[\dot{H}_c\rfloor_{GU} + W_{A,GU}{}^{\star} H_c\rfloor_{GU}\right] + W_{IA}{}^{\star} Q_{A,GU} H_c\rfloor_{GU}$$

(Eq. E. 1. 1)

Let $Q_{VA} = I$

$$Q_{A,GU} = \begin{bmatrix} C\gamma & -S\gamma & 0 \\ S\gamma & C\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This assumes Controller Position 1

$$W_{IA} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad \text{and} \quad \dot{W}_{IA} = \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}$$

(Eq. E. 1. 2)

$$W_{A,GU} = \begin{bmatrix} 0 \\ 0 \\ \dot{\gamma} \end{bmatrix}$$

$$H_c\rfloor_{GU} = J_c\, W_{I,GU}$$

(Eq. E. 1. 3)

$$\dot{H}_c\Big]_{GU} = J_c \left\{ Q_{GU,A} \left[\dot{W}_{IA} + W_{GU,A} \star W_{IA} \right] + \dot{W}_{A,GU} \right\}$$

(Eq. E.1.4)

Substituting these matrices and performing the indicated operations gives for the contribution of the case terms the following moments.

Case Terms

$$
\sum M_{\text{case 1}}\Big]_A = \Big[J_{cx}\left[(p\,C\gamma + q\,S\gamma)\,(-r\,S\gamma - \dot{\gamma}\,S\alpha) + C^2\gamma\,(\dot{p} + q\,\dot{\gamma}) + C\gamma S\gamma\,(\dot{q} - p\,\dot{\gamma})\right] +
$$

$$
J_{cx}\left[(p\,C\gamma + q\,S\gamma)\,(r\,C\gamma + \dot{\gamma}\,C\gamma) + S\gamma C\gamma\,(\dot{p} + q\,\dot{\gamma}) + S^2\gamma\,(\dot{q} - p\,\dot{\gamma})\right] +
$$

$$
J_{cx}\Big[(p\,C\gamma + q\,S\gamma)\,(p\,S\gamma - q\,C\gamma) +
$$

$$
+ J_{cy}\left[(-p\,S\gamma + q\,C\gamma)\,(-r\,C\gamma - \dot{\gamma}\,C\gamma) + S^2\gamma\,(\dot{p} + q\,\dot{\gamma}) - S\gamma C\gamma\,(\dot{q} - p\,\dot{\gamma})\right]
$$

$$
+ J_{cy}\left[(-p\,S\gamma + q\,C\gamma)\,(-r\,S\gamma - \dot{\gamma}\,S\gamma) - S\gamma C\gamma\,(\dot{p} + q\,\dot{\gamma}) + C^2\gamma\,(\dot{q} - p\,\dot{\gamma})\right]
$$

$$
+ J_{cy}\left[(-p\,S\gamma + q\,C\gamma)\,(p\,C\gamma + q\,S\gamma)\right]
$$

$$
\left.\begin{aligned}
&+ J_{cz}\,(r + \dot{\gamma})\,q \\
&- J_{cz}\,(r + \dot{\gamma})\,p \\
&+ J_{cz}\,(\ddot{\gamma} + \dot{r})
\end{aligned}\right]
$$

(Eq. E.1.5)

E. 2 Gimbal Terms

By an identical procedure as the previous section, the gimbal terms are found to be the following.

Gimbal Terms

$$\sum M_g \rfloor_A = Q_{A,\,GIM} \left[\dot{H}_g \rfloor_{GIM} + W_{A,\,GIM}{}^{\star} H_g \rfloor_{GIM} \right] + W_{IA}{}^{\star} Q_{A,\,GIM}\, H_g \rfloor_{GIM} = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}$$

where

$$\begin{aligned} A_1 = J_{gx} \Big\{ & -S\gamma\,(r + \dot{\gamma})\,(pC\gamma + qS\gamma + \dot{\alpha}) \\ & + C\gamma \left[\ddot{\alpha} + C\gamma\,(\dot{p} + q\,\dot{\gamma}C\alpha + r\dot{\gamma}S\alpha) + S\gamma\,(\dot{q} - p\dot{\gamma}C\alpha + r\dot{\alpha}) \right] \Big\} \\ + J_{gy} \Big\{ & \left[-(r + \dot{\gamma})\,C\alpha\,C\gamma + (q + \dot{\alpha}S\gamma)\,S\alpha \right] \left[-p\,S\gamma\,C\alpha + q\,C\alpha\,C\gamma + r\,S\alpha + \dot{\gamma}S\alpha \right] \\ & - S\gamma C\alpha \left[\ddot{\gamma}S\alpha + \dot{\gamma}\alpha C\alpha - S\gamma C\alpha\,(\dot{p} + q\dot{\gamma}C\alpha - r\dot{\gamma}S\gamma) + C\alpha C\gamma\,(\dot{q} - p\dot{\gamma}C\alpha + r\dot{\alpha} + S\alpha\,(\dot{r} + p\dot{\gamma}S\alpha - q\dot{\alpha}) \right] \Big\} \\ + J_{gz} \Big\{ & \left[(r + \dot{\gamma})\,S\alpha\,C\gamma + (q + \dot{\alpha}\,C\gamma)\,C\alpha \right] \left[p\,S\gamma\,S\alpha - q\,C\gamma\,S\alpha + r\,C\alpha + \dot{\gamma}\,C\alpha \right] \\ & + S\gamma S\alpha \left[\ddot{\gamma}C\alpha - \dot{\gamma}\dot{\alpha}\,S\alpha + S\gamma\,S\alpha\,(\dot{p} + q\dot{\gamma}\,C\alpha - r\dot{\gamma}S\gamma) - C\gamma\,S\alpha\,(\dot{q} - p\dot{\gamma}\,C\alpha + r\dot{\alpha}) + C\alpha\,(\dot{r} + p\dot{\gamma}\,S\alpha - q\dot{\alpha}) \right] \Big\} \end{aligned}$$

$$
\begin{aligned}
A_2 = J_{gx} \Big\{ & C\gamma (r + \dot{\gamma}) \, (p\,C\gamma + q\,S\gamma + \dot{\alpha}) \\
& + S\gamma \Big[\ddot{\alpha} + C\gamma (\dot{p} + q\dot{\gamma}\,C\alpha + r\dot{\gamma}S\gamma) + S\gamma \, (\dot{q} - p\dot{\gamma}\,C\alpha + r\dot{\alpha}) \Big] \Big\} \\
& + J_{gy} \Big\{ \Big[-(r + \dot{\gamma})\, C\alpha S\gamma - (p + \dot{\alpha}\,C\gamma)\, S\alpha \Big] \Big[-p\,S\gamma\,C\alpha + q\,C\alpha\,C\gamma + r\,S\alpha + \dot{\gamma}\,S\alpha \Big] \\
& \quad + C\alpha C\gamma \Big[\ddot{\gamma}S\alpha + \dot{\gamma}\dot{\alpha}C\alpha - S\gamma C\alpha (\dot{p} + q\dot{\gamma}C\alpha - r\dot{\gamma}S\gamma) + C\alpha C\gamma (\dot{q} - p\dot{\gamma}C\alpha + r\dot{\alpha}) + S\alpha (\dot{r} + p\dot{\gamma}S\alpha - q\dot{\alpha}) \Big] \Big\} \\
& + J_{gz} \Big\{ \Big[(r + \dot{\gamma})\, S\alpha\, S\gamma - (p + \dot{\alpha}\,C\gamma)\, C\alpha \Big] \Big[p\,S\gamma S\alpha - q\,C\gamma S\alpha + r\,C\alpha + \dot{\gamma}\,C\alpha \Big] \\
& \quad - C\gamma S\alpha \Big[\ddot{\gamma}C\alpha - \dot{\gamma}\dot{\alpha}S\alpha + S\gamma S\alpha (\dot{p} + q\dot{\gamma}C\alpha - r\dot{\gamma}S\alpha) - C\gamma S\alpha (\dot{q} - p\dot{\gamma}C\alpha + r\dot{\alpha}) + C\alpha (\dot{r} + p\dot{\gamma}S\alpha - q\dot{\alpha}) \Big] \Big\}
\end{aligned}
$$

$$A_3 = J_{gx} \Big\{ (p\,S\gamma - q\,C\gamma)\,(p\,C\gamma + q\,S\gamma + \dot{\alpha}) \Big\}$$

$$+ J_{gy} \Big\{ C\alpha \left[\dot{\alpha} + p\,C\gamma + q\,S\gamma\right] \left[-p\,S\gamma\,C\alpha + q\,C\alpha\,C\gamma + r\,S\alpha + \dot{\gamma}\,S\alpha\right]$$

$$+ S\alpha \left[\ddot{\gamma}S\alpha + \dot{\gamma}\alpha C\alpha - S\gamma C\alpha\,(\dot{p} + q\dot{\gamma}C\alpha - r\dot{\gamma}S\gamma) + C\alpha C\gamma\,(\dot{q} - p\dot{\gamma}C\alpha + r\dot{\alpha}) + S\alpha\,(\dot{r} + p\dot{\gamma}S\alpha - q\dot{\alpha})\right]\Big\}$$

$$+ J_{gz} \Big\{ S\alpha \left[-\dot{\alpha} - p\,C\gamma - q\,S\gamma\right] \left[p\,S\gamma\,S\alpha - q\,C\gamma\,S\alpha + r\,C\alpha + \dot{\gamma}\,C\alpha\right]$$

$$+ C\alpha \left[\ddot{\gamma}C\alpha - \dot{\gamma}\dot{\alpha}S\alpha + S\gamma S\alpha\,(\dot{p} + q\dot{\gamma}C\alpha - r\dot{\gamma}S\gamma) - C\gamma S\alpha\,(\dot{q} - p\dot{\gamma}C\alpha + r\dot{\alpha}) + C\alpha\,(\dot{r} + p\dot{\gamma}S\alpha - q\dot{\alpha})\right]\Big\}$$

(Eq. E. 2. 1)

Above is adequate when two of the three attitude angles are small. Otherwise we must let

$$p = \dot{\phi} - \dot{\psi}\,S\theta$$

$$q = \dot{\theta}\,C\phi - \dot{\psi}\,S\phi\,C\theta$$

$$r = \dot{\psi}\,C\phi\,C\theta - \dot{\theta}\,S\phi$$

The preceding is adequate when two of the three attitude angles are small, otherwise we must substitute

$$p = \dot{\phi} - \dot{\psi}\, S\theta$$

$$q = \dot{\theta}\, C\phi - \dot{\psi}\, S\phi C\theta$$

$$r = \dot{\psi}\, C\phi C\theta - \dot{\theta}\, S\phi$$

E. 3 Rotor Terms

By an identical procedure as in section E. 1 the exact rotor terms for a rotor of a controller in position 1 are found to be the following.

Define

J_{rd} = dimetrical moment of inertia of rotor

J_{rp} = polar moment of inertia of rotor

The following equations are adequate when two of the three attitude angles are small. Otherwise we must substitute

$$p = \dot{\phi} - \dot{\psi} S\theta$$

$$q = \dot{\theta} C\phi - \dot{\psi} S\phi C\theta$$

$$r = \dot{\psi} C\phi C\theta - \dot{\theta} S\phi$$

Rotor Terms

$$\sum M_{rotor}\Big]_A = \begin{bmatrix} C\gamma & -S\gamma C\alpha & +S\gamma S\alpha \\ S\gamma & C\gamma C\alpha & -C\gamma S\alpha \\ 0 & S\alpha & C\alpha \end{bmatrix} \begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{h}_3 \end{bmatrix}$$

$$+ \begin{bmatrix} -(r + \dot{\gamma})\, S\gamma\alpha & \left[-(r + \dot{\gamma})\, C\alpha C\gamma + (q + \dot{\alpha} S\gamma)\, S\alpha\right] & \left[(r\, S\alpha + \dot{\gamma})\, C\gamma + (q + \dot{\alpha} S\gamma)\, C\alpha\right] \\ (r + \dot{\gamma})\, C\gamma & \left[-(r + \dot{\gamma})\, C\alpha S\gamma - (p + \dot{\alpha} C\gamma)\, S\alpha\right] & \left[(r\, S\alpha + \dot{\gamma})\, S\gamma - (p + \dot{\alpha} C\gamma)\, C\alpha\right] \\ (pS\gamma - qS\gamma) & \left[pC\gamma + qS\gamma + \dot{\alpha}\right] C\alpha & \left[-pC\gamma - qS\gamma - \dot{\alpha}\right] S\alpha \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

where

$$h_1 = J_{rd} \left\{ p\, C\gamma + q\, S\gamma + \dot{\alpha} \right\}$$

$$h_2 = J_{rp} \left\{ -p\, C\alpha S\gamma + q\, C\alpha C\gamma + r\, S\alpha + \dot{\gamma} S\alpha + \dot{\beta} \right\}$$

$$h_3 = J_{rd} \left\{ p\, S\alpha S\gamma - q\, S\alpha C\gamma + r\, C\alpha + \dot{\gamma} C\alpha \right\}$$

$$\dot{h} = J_{rd} \left\{ (q\, C\gamma - p\, S\gamma)\, \dot{\gamma} + \ddot{\alpha} \right\}$$

$$\dot{h}_2 = J_{rp} \left\{ (-p\, C\gamma - q\, S\gamma)\, C\alpha\dot{\gamma} + (p\, S\gamma - q\, C\gamma)\, S\alpha\dot{\alpha} + (r + \dot{\gamma})\, C\alpha\dot{\alpha} + \ddot{\gamma} S\alpha + \ddot{\beta} \right\}$$

$$\dot{h}_3 = J_{rd} \left\{ (p\, C\gamma + q\, S\gamma)\, S\alpha\dot{\gamma} + (p\, S\gamma - q\, C\gamma)\, C\alpha\dot{\alpha} - (r + \dot{\gamma})\, S\alpha\dot{\alpha} + \ddot{\gamma} C\alpha \right\} \qquad \text{(Eq. E. 3. 1)}$$

APPENDIX F

APPROXIMATE EQUATIONS FOR CONTROLLERS

F. 1 Arbitrary Controller

Using Equation 2. 3. 8 and assuming the angular momentum of the rotor is predominately along the spin axis of the rotor, [33] one can derive the approximate equation

$$\sum M_{CS}\Big]_A = Q_{A,\,GIM}\, W_{A,\,GIM}^{\star}\, H_r\Big]_{GIM} + W_{IA}^{\star}\, Q_{A,\,GIM}\, H_r\Big]_{GIM} \qquad \text{(Eq. F. 1. 1)}$$

where $$H_r\Big]_{GIM} = \begin{bmatrix} 0 \\ J_P\,\dot{\beta} \\ 0 \end{bmatrix} \qquad \text{(Eq. F. 1. 2)}$$

Now $$W_{IA} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad \text{from Appendix D} \qquad \text{(Eq. F. 1. 3)}$$

$$Q_{A,\,GIM} = Q_{A,\,GU}\, Q_{GU,\,GIM} \qquad \text{(Eq. F. 1. 4)}$$

and can be found from the coordinate transformations of Appendix C. 1

$$W_{A,\,GIM} = Q_{GIM,\,GU}\, W_{A,\,GU} + W_{GU,\,GIM} \qquad \text{(Eq. F. 1. 5)}$$

and can likewise be found using matrices of Appendix C and Appendix D.

Note that equation F. 1. 1 is a sum of two parts. The first of these parts will form the primary control matrix and the second part will form the gyroscopic coupling matrix of the final equation in a particular system using one or more of the controllers.

After performing the indicated operations, the equation for a single controller with an arbitrary case position is given by the following equation.

$$
\begin{aligned}
\sum M_{CS}\Big]_A = J_P \dot{\beta} \Bigg[& \Big\{ \dot{A}_{YU} \left(S\alpha\, CA_{X,GU} + C\alpha\, SA_{X,GU} \right) + \gamma \left(S\alpha\, SA_{X,GU}\, CA_{YU} - C\alpha\, CA_{X,GU}\, CA_{YU} \right) \Big\} CA_{YU}\, C\gamma \\
& \Big\{ \dot{A}_{YU} \left(S\alpha\, CA_{X,GU} + C\alpha\, SA_{X,GU} \right) + \gamma \left(S\alpha\, SA_{X,GU}\, CA_{YU} - C\alpha\, CA_{X,GU}\, CA_{YU} \right) \Big\} CA_{YU}\, S\gamma \\
& \Big\{ \dot{A}_{YU} \left(-S\alpha\, CA_{X,GU} - C\alpha\, SA_{X,GU} \right) + \gamma \left(-S\alpha\, SA_{X,GU}\, CA_{YU} + C\alpha\, CA_{X,GU}\, CA_{YU} \right) \Big\} SA_{YU} \\
& + \left(\dot{A}_{X,GU} - SA_{YU}\, \dot{\gamma} + \dot{\alpha} \right) \Big(S\alpha \left[CA_{X,GU}\, S\gamma - SA_{X,GU}\, SA_{YU}\, C\gamma \right] + C\alpha \left[SA_{X,GU}\, S\gamma + CA_{X,GU}\, SA_{YU}\, C\gamma \right] \Big) \\
& + \left(\dot{A}_{X,GU} - SA_{YU}\, \dot{\gamma} + \dot{\alpha} \right) \Big(S\alpha \left[-CA_{X,GU}\, C\gamma - SA_{X,GU}\, SA_{YU}\, S\gamma \right] + C\alpha \left[-SA_{X,GU}\, C\gamma + CA_{X,GU}\, SA_{YU}\, S\gamma \right] \Big) \\
& + \left(\dot{A}_{X,GU} - SA_{YU}\, \dot{\gamma} + \dot{\alpha} \right) \Big(-S\alpha\, SA_{X,GU}\, CA_{YU} + C\alpha\, CA_{X,GU}\, CA_{YU} \Big) \Bigg] + \\
+ J_P \dot{\beta} \Bigg[& - r \left[C\alpha \left(CA_{X,GU}\, C\gamma + SA_{X,GU}\, SA_{YU}\, S\gamma \right) + S\alpha \left(-SA_{X,GU}\, C\gamma + CA_{X,GU}\, SA_{YU}\, S\gamma \right) \right] \\
& + r \left[C\alpha \left(-CA_{X,GU}\, S\gamma + SA_{X,GU}\, SA_{YU}\, C\gamma \right) + S\alpha \left(SA_{X,GU}\, S\gamma + CA_{X,GU}\, SA_{YU}\, C\gamma \right) \right] \\
& - q \left[C\alpha \left(-CA_{X,GU}\, S\gamma + SA_{X,GU}\, SA_{YU}\, C\gamma \right) + S\alpha \left(SA_{X,GU}\, S\gamma + CA_{X,GU}\, SA_{YU}\, C\gamma \right) \right] \Bigg] \\
& + q \left[SA_{X,GU}\, CA_{YU}\, C\alpha + CA_{X,GU}\, CA_{YU}\, S\alpha \right] \\
& - p \left[SA_{X,GU}\, CA_{YU}\, C\alpha + CA_{X,GU}\, CA_{YU}\, S\alpha \right] \\
& + p \left[C\alpha \left(CA_{X,GU}\, C\gamma + SA_{X,GU}\, SA_{YU}\, S\gamma \right) + S\alpha \left(-SA_{X,GU}\, C\gamma + CA_{X,GU}\, SA_{YU}\, S\gamma \right) \right] \Bigg]
\end{aligned}
$$

(Eq. F. 1. 6)

It is not considered desirable to put the preceding equation in matrix form as there would be no simplification. However, upon use of the preceding equation in a particular controller position, the time rates of the case angles will usually vanish and considerable simplification then results. At this point it is desirable to put the remaining terms in matrix form to assist their summation with terms from other controllers with the same control system input variables.

F. 2 Equations for Inertia Wheels

Unfortunately equation F. 1. 6 cannot be used for the case of inertia reaction wheels because it has been assumed in that section that the angular velocity of the controller is a large value and is constant. Equation E. 3. 1 can be used for the X wheel and the Y wheel or any other wheel that lies in the x-y principle axis frame or for a pure z-aligned wheel. For the perfectly arbitrary inertia reaction wheel it may be desirable to list here a general expression for the moments. If we consider that the wheels are rigidly mounted in the spacecraft then equation 2. 3. 8 can be used and is written as follows.

$$\sum M_{CS}\Big]_A = \sum_{i=1}^{N} \left\{ Q_{AR} \left[\dot{H}_r \Big]_R + W_{AR}{}^{\star} H_r \Big]_R \right] + W_{IA}{}^{\star} Q_{AR} \, H_r \Big]_R \right\}_i \qquad \text{(Eq. F. 2. 1)}$$

Or, the same equation written with respect to the case frame is

$$\sum M_{CS}\Big]_A = \sum_{i=1}^{N} \left\{ Q_{A,GU} \left[\dot{H}_r \Big]_{GU} + W_{A,GU}{}^{\star} H_r \Big]_{GU} \right] + W_{IA}{}^{\star} Q_{A,GU} \, H_r \Big]_{GU} \right\}_i \qquad \text{(Eq. F. 2. 2)}$$

Since the case holding the wheel is rigidly attached to the spacecraft $W_{A,GU}{}^{\star} = 0$

$$\sum M_{CS}\Big]_A = \sum_{i=1}^{N} \left\{ Q_{A,GU} \, \dot{H}_r \Big]_{GU} + W_{IA}{}^{\star} Q_{A,GU} \, H_r \Big]_{GU} \right\}_i$$

(Eq. F. 2. 2)

The most difficult part of this derivation is that of finding $H_r\rfloor_{GU}$. A considerable simplification of the algebra results if we agree in this section to let

$$Q_{GU,A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & m & n \end{bmatrix} \qquad \text{(Eq. F. 2. 3)}$$

Where the elements of equation F. 2. 3 are given by equation C. 1. 7 for the case where the vehicle reference frame is identical with the principle axis frame. Then it is determined that the angular momentum with respect to the case frame is as follows.

$$H_r\rfloor_{GU} = J_r\rfloor_{GU} Q_{GU,A} W_{IA} + J_r\rfloor_{GU} Q_{GU,R} W_{GIM,R}$$

$$H_r\rfloor_{GU} = \begin{bmatrix} J_X a & J_X b & J_X c \\ J_P d & J_P e & J_P f \\ J_X g & J_X m & J_X n \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \begin{bmatrix} 0 \\ J_P \dot{\beta} \\ 0 \end{bmatrix}$$

(Eq. F. 2. 4)

where

$$J_r\rfloor_R = \begin{bmatrix} J_X & 0 & 0 \\ 0 & J_P & 0 \\ 0 & 0 & J_X \end{bmatrix} = J_r\rfloor_{GU}$$

(Eq. F. 2. 5)

Note that $\mathring{H}_r\rfloor_{GU} = J_r\rfloor_{GU} Q_{GU,A} \mathring{W}_{IA} + J_r\rfloor_{GU} Q_{GU,R} \dot{W}_{GIM,R}$

$$\dot{H}_r \Big]_{GU} = \begin{bmatrix} J_X a & J_X b & J_X c \\ J_P d & J_P e & J_P f \\ J_X g & J_X m & J_X n \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} 0 \\ J_P \ddot{\beta} \\ 0 \end{bmatrix}$$

(Eq. F. 2. 6)

Following through the indicated operations by substituting the above matrices into equation F. 2. 2 gives the following equation for the moments contributed by the inertia reaction wheels of an attitude control system.

$$\sum M_{CS}\Big]_A = \sum_{i=1}^{N} \left\{ J_P \ddot{\beta} \begin{bmatrix} d \\ e \\ f \end{bmatrix} + J_P \dot{\beta} \begin{bmatrix} 0 & f & -e \\ -f & 0 & d \\ e & -d & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right\}_i$$

$$+ \sum_{i=1}^{N} \left\{ Q_{A,\,GU} \begin{bmatrix} J_X a & J_X b & J_X c \\ J_P d & J_P e & J_P f \\ J_X g & J_X m & J_X m \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} \right.$$

$$\left. + W_{IA} \star Q_{A,\,GU} \begin{bmatrix} J_X a & J_X b & J_X c \\ J_P d & J_P e & J_P f \\ J_X g & J_X m & J_X n \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right\}_i$$

(Eq. F. 2. 7)

Equation F. 2. 7 has been written in two parts because it is convenient to lump the second part with the vehicle since that part is not dependent on the control variable β. The second part relates

to the moments contributed to the vehicle by the wheels acting as inert masses constrained to move with the vehicle. Therefore the equation is written as the following.

$$\sum M_{CS}\Big]_A = \sum_{i=1}^{N} \left\{ J_P \ddot{\beta} \begin{bmatrix} d \\ e \\ f \end{bmatrix} + J_P \dot{\beta} \begin{bmatrix} 0 & f & -e \\ -f & 0 & d \\ e & -d & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right\}_i$$

(Eq. F. 2. 8)

where $d = -CA_{X,GU}\, S\gamma + SA_{X,GU}\, SA_{YU}\, C\gamma$

$e = CA_{X,GU}\, C\gamma + SA_{X,GU}\, SA_{YU}\, S\gamma$

$f = SA_{X,GU}\, CA_{YU}$

from equation F. 2. 3 and C. 1. 7

	d	e	f
For an X-wheel	1	0	0
Y-wheel	0	1	0
Z-wheel	0	0	1

<u>For an X-wheel</u>

$$\sum M_{X\text{-wheel}}\Big]_A = \begin{bmatrix} J_P \ddot{\beta} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & J_P \dot{\beta} \\ 0 & -J_P \dot{\beta} & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. F. 2. 9)

For a Y-wheel

$$\sum M_{Y\text{-wheel}}\Big]_A = \begin{bmatrix} 0 \\ J_P\ddot{\beta} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -J_P\dot{\beta} \\ 0 & 0 & 0 \\ J_P\dot{\beta} & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. F. 2. 10)

For a Z-wheel

$$\sum M_{Z+\text{wheel}}\Big]_A = \begin{bmatrix} 0 \\ 0 \\ J_P\ddot{\beta} \end{bmatrix} + \begin{bmatrix} 0 & J_P\dot{\beta} & 0 \\ -J_P\dot{\beta} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. F. 2. 11)

F. 3 Equations for a Particular Three Degree-of-Freedom Controller

Only one controller is considered wherein three degrees-of-freedom of the rotor are allowed. This controller is aligned with its spin reference axis along the positive x-axis of the vehicle-centered principal axis frame. Thus in equation E. 3. 1 if we let $\gamma = \gamma - 90^{\circ}$ and $J_{r_d} = 0$, the following equation is obtained

$$\sum M_{(3-3-3)}\Big]_A = J_{rp} \begin{bmatrix} C\alpha C\gamma & -S\gamma C\alpha & -C\gamma S\alpha \\ S\gamma C\alpha & C\alpha C\gamma & -S\gamma S\alpha \\ S\alpha & 0 & C\alpha \end{bmatrix} \begin{bmatrix} \ddot{\beta} \\ \dot{\beta}\dot{\gamma} \\ \dot{\beta}\dot{\alpha} \end{bmatrix}$$

$$+ J_{rp}\dot{\beta} \begin{bmatrix} 0 & S\alpha & -C\alpha S\gamma \\ -S\alpha & 0 & C\alpha C\gamma \\ S\gamma C\alpha & -C\gamma C\alpha & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. F. 3. 1)

F. 4 Equations for Particular Two-Degree-of-Freedom Controllers

In the synthesis of the moment equations for those control systems that are composed of several controllers of the two-degree-of-freedom type, it is convenient to have the simplified moment equations of each controller. Thus the controllers defined by the following angles are presented. See Figure B. 9 and paragraphs A-4 and C-3.

TABLE F. 4

POSITION ANGLES FOR TDF CONTROLLERS

TDF CONTROLLER	CASE ANGLES		
	$A_{X, GU}$	A_{YU}	γ
1	0	0	$0 + \gamma$
2	0	0	$\pm 180^{\circ} + \gamma$
3	0	0	$-90^{\circ} + \gamma$
4	0	0	$+90^{\circ} + \gamma$
7	0	0	$+120^{\circ} + \gamma$
8	0	0	$-120^{\circ} + \gamma$

If the above reference angles are substituted into equation F. 1. 6 the following equations are obtained. Note that γ is a case angle retained as a control input variable.

Controller 1

$$\sum M_{TDF\,1}\Big]_A = J_P\dot{\beta}\begin{bmatrix} -C\alpha C\gamma & 0 & +S\alpha S\gamma \\ -C\alpha S\gamma & 0 & -S\alpha C\gamma \\ 0 & 0 & +C\alpha \end{bmatrix}\begin{bmatrix} \dot{\gamma} \\ 0 \\ \dot{\alpha} \end{bmatrix} + J_P\dot{\beta}\begin{bmatrix} 0 & +S\alpha & -C\alpha C\gamma \\ -S\alpha & 0 & -S\gamma C\alpha \\ C\alpha C\gamma & +C\alpha S\gamma & 0 \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. F. 4. 1)

Controller 2

$$\sum M_{TDF\,2}\Big]_A = J_P\dot{\beta}\begin{bmatrix} C\alpha C\gamma & 0 & -S\alpha S\gamma \\ C\alpha S\gamma & 0 & +S\alpha C\gamma \\ 0 & 0 & C\alpha \end{bmatrix}\begin{bmatrix} \dot{\gamma} \\ 0 \\ \dot{\alpha} \end{bmatrix} + J_P\dot{\beta}\begin{bmatrix} 0 & +S\alpha & +C\alpha C\gamma \\ -S\alpha & 0 & +S\gamma C\alpha \\ -C\alpha C\gamma & -S\gamma C\alpha & 0 \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. F. 4. 2)

Controller 3

$$\sum M_{TDF\,3}\Big]_A = J_P\dot{\beta}\begin{bmatrix} 0 & -C\alpha S\gamma & -S\alpha C\gamma \\ 0 & +C\alpha C\gamma & -S\alpha S\gamma \\ 0 & 0 & C\alpha \end{bmatrix}\begin{bmatrix} 0 \\ \dot{\gamma} \\ \dot{\alpha} \end{bmatrix} + J_P\dot{\beta}\begin{bmatrix} 0 & +S\alpha & -C\alpha S\gamma \\ -S\alpha & 0 & +C\gamma C\alpha \\ +C\alpha S\gamma & -C\alpha C\gamma & 0 \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. F. 4. 3)

Controller 4

$$\sum M_{TDF\,4}\Big|_A = J_P\dot{\beta}\begin{bmatrix}0 & C\alpha S\gamma & +S\alpha C\gamma\\ 0 & -C\alpha C\gamma & +S\alpha S\gamma\\ 0 & 0 & C\alpha\end{bmatrix}\begin{bmatrix}0\\ \dot{\gamma}\\ \dot{\alpha}\end{bmatrix} + J_P\dot{\beta}\begin{bmatrix}0 & +S\alpha & +C\alpha S\gamma\\ -S\alpha & 0 & -C\gamma C\alpha\\ -C\alpha S\gamma & +C\alpha C\gamma & 0\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix}$$

(Eq. F. 4. 4)

Controller 7

$$\sum M_{TDF\,7}\Big|_A = J_P\dot{\beta}\begin{bmatrix}C\alpha C(\gamma-60^o) & 0 & -S\alpha S(\gamma-60^o)\\ +C\alpha S(\gamma-60^o) & 0 & +S\alpha C(\gamma-60^o)\\ 0 & 0 & C\alpha\end{bmatrix}\begin{bmatrix}\dot{\gamma}\\ 0\\ \dot{\alpha}\end{bmatrix}$$

$$+ J_P\beta\begin{bmatrix}0 & +S\alpha & C\alpha C(\gamma-60^o)\\ -S\alpha & 0 & C\alpha S(\gamma-60^o)\\ -C\alpha C(\gamma-60^o) & -C\alpha S(\gamma-60^o) & 0\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix}$$

(Eq. F. 4. 5)

Controller 8

$$\sum M_{TDF\,8}\Big]_A = J_P\dot{\beta}\begin{bmatrix} C\alpha\, C(\gamma+60^{\circ}) & 0 & -S\alpha\, S(\gamma+60^{\circ}) \\ C\alpha\, S(\gamma+60^{\circ}) & 0 & +S\alpha\, C(\gamma+60^{\circ}) \\ 0 & 0 & C\alpha \end{bmatrix}\begin{bmatrix} \dot{\gamma} \\ 0 \\ \dot{\alpha} \end{bmatrix}$$

$$+ J_P\dot{\beta}\begin{bmatrix} 0 & S\alpha & C\alpha\, C(\gamma+60^{\circ}) \\ -S\alpha & 0 & C\alpha\, S(\gamma+60^{\circ}) \\ -C\alpha\, C(\gamma+60^{\circ}) & -C\alpha\, S(\gamma+60^{\circ}) & 0 \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. F. 4. 6)

F. 5 Equations for Particular Single-Degree-of-Freedom Controllers

In the synthesis of the moment equations for those control systems that are composed of several controllers of the single-degree-of-freedom type, it is convenient to have available the simplified equations of each controller. The following table of angles define the orientation of the case relative to the vehicle centered principal axis frame. See Figure B. 9.

TABLE F. 5

POSITION ANGLES FOR SDF CONTROLLERS

SDF CONTROLLER	CASE ANGLES		
	$A_{X, GU}$	A_{YU}	γ
1	0	0	0
2	0	0	$\pm 180^o$
3	0	-90^o	-90^o
4	0	$+90^o$	$+90^o$
5	$+90^o$	0	-90^o
6	-90^o	0	$+90^o$
7	0	-90^o	$+120^o$
8	0	-90^o	-120^o
9	$-\alpha_o$	0	$\pm 180^o$
10	$+\alpha_o$	0	$\pm 180^o$

SINGLE-DEGREE-OF-FREEDOM CONTROLLERS

(Subscripts of Variables Omitted)*

Controller 1

$$\sum M_{SDF\,1}\Big]_A = J_P \dot{\beta} \begin{bmatrix} q\,S\alpha & - & r\,C\alpha \\ -\dot{\alpha}\,S\alpha & - & p\,S\alpha \\ +\dot{\alpha}\,C\alpha & + & p\,C\alpha \end{bmatrix} \qquad \text{(Eq. F. 5. 1)}$$

Controller 2

$$\sum M_{SDF\,2}\Big]_A = J_P \dot{\beta} \begin{bmatrix} q\,S\alpha & + & r\,C\alpha \\ -p\,S\alpha & + & \dot{\alpha}\,S\alpha \\ -p\,C\alpha & + & \dot{\alpha}\,C\alpha \end{bmatrix} \qquad \text{(Eq. F. 5. 2)}$$

Controller 3

$$\sum M_{SDF\,3}\Big]_A = J_P \dot{\beta} \begin{bmatrix} -(r + \dot{\alpha})\,S\alpha \\ +(r + \dot{\alpha})\,C\alpha \\ p\,S\alpha - q\,C\alpha \end{bmatrix} \qquad \text{(Eq. F. 5. 3)}$$

Controller 4

$$\sum M_{SDF\,4}\Big]_A = J_P \dot{\beta} \begin{bmatrix} (\dot{\alpha} - r)\,S\alpha \\ (\dot{\alpha} - r)\,C\alpha \\ p\,S\alpha + q\,C\alpha \end{bmatrix} \qquad \text{(Eq. F. 5. 4)}$$

Controller 5

$$\sum M_{SDF\,5}\Big]_A = J_P \dot{\beta} \begin{bmatrix} (q - \dot{\alpha})\,C\alpha \\ -p\,C\alpha - r\,S\alpha \\ (q - \dot{\alpha})\,S\alpha \end{bmatrix} \qquad \text{(Eq. F. 5. 5)}$$

Controller 6

$$\sum M_{SDF\,6}\Big]_A = J_P \dot{\beta} \begin{bmatrix} -(q + \dot{\alpha})\, C\alpha \\ p\, C\alpha - r\, S\alpha \\ (q + \dot{\alpha})\, S\alpha \end{bmatrix} \qquad \text{(Eq. F. 5. 6)}$$

Controller 7

$$\sum M_{SDF\,7}\Big]_A = J_P \dot{\beta} \begin{bmatrix} (r + \dot{\alpha})\, S\,(30^\circ + \alpha) \\ -(r + \dot{\alpha})\, C\,(30^\circ + \alpha) \\ -p\,S\,(30^\circ + \alpha) + q\,C\,(30^\circ + \alpha) \end{bmatrix}$$

(Eq. F. 5. 7)

Controller 8

$$\sum M_{SDF\,8}\Big]_A = J_P \dot{\beta} \begin{bmatrix} (r + \dot{\alpha})\, S\,(30^\circ - \alpha) \\ (r + \dot{\alpha})\, C\,(30^\circ - \alpha) \\ -p\,S\,(30^\circ - \alpha) - q\,C\,(30^\circ - \alpha) \end{bmatrix}$$

(Eq. F. 5. 8)

Controller 9

$$\sum M_{SDF\,9}\Big]_A = J_P \dot{\beta} \begin{bmatrix} r\,C\,(\alpha_o - \alpha) - q\,S\,(\alpha_o - \alpha) \\ +(p - \alpha)\, S\,(\alpha_o - \alpha) \\ -(p - \alpha)\, C\,(\alpha_o - \alpha) \end{bmatrix}$$

(Eq. F. 5. 9)

Controller 10

$$\sum M_{SDF\,10}\Big]_A = J_P \dot{\beta} \begin{bmatrix} +q\,S\,(\alpha_o + \alpha) + r\,C\,(\alpha_o + \alpha) \\ +(\dot{\alpha} - p)\, S\,(\alpha_o + \alpha) \\ +(\dot{\alpha} - p)\, C\,(\alpha_o + \alpha) \end{bmatrix}$$

(Eq. F. 5. 10)

* Note that the equations of this section have been written in the form of a column vector because they are simple expressions with only a few terms. When a number of the above equations are added to form a complete control system, the form of Equation 3. 2. 3 is recommended.

APPENDIX G

EQUATIONS FOR PARTICULAR MOMENTUM EXCHANGE ATTITUDE CONTROL SYSTEMS

G. 1 INTRODUCTION

A spacecraft attitude control system of the momentum exchange type will usually contain a number of momentum elements either of the inertia reaction wheel or of the gyro type. In this thesis inertia reaction wheels are referred to as simply inertia wheels and have fixed axes of rotation whereas gyros are called controllers to differentiate from the conventional gyros used in inertial reference systems and in addition to its spin motion a controller may have one or more degrees of freedom. To determine the moment contributions of a particular system consisting of several controllers or wheels as the case may be, one needs only to sum the contributions of the individual controllers and wheels. A number of the more common controllers are represented in Appendix F.

In this Appendix a number of control systems have been defined by choosing various configurations of controllers. The following table indicates the controllers which constitute the system.

G. 2 List of Control Systems and their Controllers

Name of System	Type	Controller No. 1	2	3	4	5	6	7	8	9	10
Single Controller (3-3-3)	3DF			X							
Sun Pointing (0-34-34)*	TDF			X	X						
Sun Pointing (12-0-12)	TDF	X	X								
Three Controller (2-34-234)#	TDF		X	X	X						
Three Controller (1-34-134)#	TDF	X		X	X						
Three Controller (12-4-124)#	TDF	X	X		X						
Three Controller (12-3-123)#	TDF	X	X	X							
Four Controller (12-34-1234)	TDF	X	X	X	X						
Six Controller (56-34-12)	SDF	X	X	X	X	X	X				
Three Controller Orthogonal (5-3-1)	SDF	X		X		X					
Three Controller Zero Momentum (78-78-1)	SDF	X						X	X		
Three Controller Zero Momentum (178-78-178)	TDF	X						X	X		
Two Controller (0-0-12)	SDF	X	X								
Two Controller (0-34-0)	SDF			X	X						
Two Controller (56-0-0)	SDF					X	X				
MIT "Vertical Vee" (0-0-9, 10)	SDF									X	X
Sun Pointing (0-34-12)	SDF	X	X	X	X						
Three Orthogonal Inertial Wheels	Wheels	X		X		X					

* The numbers in the parenthesis partially define the control system by giving the controllers used for roll, pitch, and yaw respectively. Thus the code following the Sun Pointing System (0-34-34) indicates that there is no roll control; controllers 3 and 4 control both pitch and yaw. A controller used in only one axis indicates a single degree-of-freedom controller, and for controllers 1 through 6 a controller used for two axes indicates a two-degree-of-freedom controllers. A controller used for three axes is a three-degree-of-freedom controller.

\# These systems are adaptive versions of the (12-34-1234) four controller system.

Single Controller System (3-3-3)

Description of System: System consists of a number 3 controller which has three degrees of freedom. The rotor of the controller is free to precess in two directions plus the rotor is capable of being accelerated about its axis of symmetry.

Control Logic: No control logic is required for the single controller system since the system contains three degrees of freedom, and the control system input matrix can be nearly diagonalized by suitable arrangement of the matrix.

$+\ddot{\beta}$ gives roll to left

$+\dot{\gamma}$ gives pitch nose down

$+\dot{\alpha}$ gives yaw to left

Moment Contribution of Control System:

$$\sum M_{(3\text{-}3\text{-}3)}\Big]_A = J_P \begin{bmatrix} +C\alpha C\gamma & -S\gamma C\alpha & -C\gamma S\alpha \\ +S\gamma C\alpha & +C\alpha C\gamma & -S\gamma S\alpha \\ +S\alpha & 0 & +C\alpha \end{bmatrix} \begin{bmatrix} \ddot{\beta} \\ \dot{\beta}\dot{\gamma} \\ \dot{\beta}\dot{\alpha} \end{bmatrix}$$

$$+ J_P \dot{\beta} \begin{bmatrix} 0 & +S\alpha & -C\alpha S\gamma \\ -S\alpha & 0 & +C\alpha C\gamma \\ +C\alpha S\gamma & -C\alpha C\gamma & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 1)

Sun Pointing System (0-34-34)

Description of System: This system is a special case of the four controller system. The controllers are TDF.

Control Logic:

$$\begin{bmatrix} \gamma_3 \\ \gamma_4 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \eta_x \\ \eta_y \\ \eta_z \end{bmatrix}$$

$+\dot{\gamma}_3$ gives pitch nose down

$+\dot{\alpha}_3$ gives yaw left

Moment Contribution of Control System:

$$\sum M_{(0-34-34)}\Big]_A = 2J_P\dot{\beta} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -C\alpha C\gamma_3 & +S\alpha S\gamma_3 \\ 0 & 0 & -C\alpha \end{bmatrix} \begin{bmatrix} \dot{\eta}_x \\ \dot{\eta}_y \\ \dot{\eta}_z \end{bmatrix}$$

$$+ 2J_P\dot{\beta} \begin{bmatrix} 0 & +S\alpha & -C\alpha S\gamma_3 \\ -S\alpha & 0 & 0 \\ +C\alpha S\gamma_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 2)

The equation written in terms of control input variables is

$$\sum M_{(0-34-34)}\Big]_A = 2\,J_P\,\dot{\beta} \begin{bmatrix} 0 & 0 & 0 \\ 0 & +C\alpha\,C\gamma_3 & -S\alpha\,S\gamma_3 \\ 0 & 0 & +C\alpha \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\gamma}_3 \\ \dot{\alpha} \end{bmatrix}$$

$$+ 2\,J_P\,\dot{\beta} \begin{bmatrix} 0 & +S\alpha & -C\alpha S\gamma_3 \\ -S\alpha & 0 & 0 \\ +C\alpha S\gamma_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 3)

Sun Pointing System (12-0-12)

Description of System: This system is a special case of the four controller system. The controllers are TDF.

Control Logic:

$$\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} +1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \eta_x \\ \eta_y \\ \eta_z \end{bmatrix}$$

$+\dot{\gamma}_1$ gives roll right

$+\dot{\alpha}_1$ gives yaw left

Moment Contribution of Control System:

$$\sum M_{(12-0-12)}\Big]_A = 2\,J_P\dot{\beta} \begin{bmatrix} -C\alpha\, C\gamma_1 & 0 & -S\alpha\, S\gamma_1 \\ 0 & 0 & 0 \\ 0 & 0 & -C\alpha \end{bmatrix} \begin{bmatrix} \dot{\eta}_x \\ \dot{\eta}_y \\ \dot{\eta}_z \end{bmatrix}$$

$$+ 2\,J_P\dot{\beta} \begin{bmatrix} 0 & S\alpha & 0 \\ -S\alpha & 0 & -C\alpha\, S\gamma_1 \\ 0 & C\alpha\, S\gamma_1 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 4)

The equation written in terms of input control variables is

$$\sum M_{(12\text{-}0\text{-}12)}\Big]_A = 2 J_P \dot{\beta} \begin{bmatrix} C\alpha C\gamma_1 & 0 & S\alpha S\gamma_1 \\ 0 & 0 & 0 \\ 0 & 0 & C\alpha \end{bmatrix} \begin{bmatrix} \dot{\gamma}_1 \\ 0 \\ \dot{\alpha} \end{bmatrix}$$

$$+ 2 J_P \dot{\beta} \begin{bmatrix} 0 & S\alpha & 0 \\ -S\alpha & 0 & -C\alpha S\gamma_1 \\ 0 & C\alpha S\gamma_1 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 5)

Three Controller System (2-34-234)

Description of System: This system is a result of failure of the number one controller in the four controller system (12-34-1234).

Control Logic: Unless the control logic is changed following the failure of the number one controller, it will be the same as the four controller system (12-34-1234). $\gamma_3 = -\gamma_4 \quad \alpha_2 = \alpha_3 = \alpha_4$

Moment Contribution of Control System:

$$\sum M_{(2-34-234)}\Big]_A = J_P \dot{\beta} \begin{bmatrix} C\alpha C\gamma_2 & 0 & -S\alpha S\gamma_2 \\ C\alpha S\gamma_2 & +2\,C\alpha C\gamma_3 & -2\,S\alpha S\gamma_3 + S\alpha C\gamma_3 \\ 0 & 0 & 3\,C\alpha \end{bmatrix} \begin{bmatrix} \dot{\gamma}_2 \\ \dot{\gamma}_3 \\ \dot{\alpha} \end{bmatrix}$$

$$+ J_P \dot{\beta} \begin{bmatrix} 0 & 3\,S\alpha & -\left(2 C\alpha S\gamma_3 - C\alpha C\gamma_2\right) \\ -3\,S\alpha & 0 & S\gamma_2\, C\alpha \\ \left(2\,C\alpha S\gamma_3 - C\alpha C\gamma_2\right) & -C\alpha\, S\gamma_2 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 6)

Three Controller System (1-34-134)

Description of System: Same as four controller system except number two controller has failed.

Control Logic: Same as four controller system. $\alpha_1 = \alpha_3 = \alpha_4$ and $\gamma_3 = -\gamma_4$

Moment Contribution of Control System:

$$\sum M_{(1-34-134)}\Big]_A = J_P \dot{\beta} \begin{bmatrix} -C\alpha C\gamma_1 & 0 & +S\alpha S\gamma_1 \\ -C\alpha S\gamma_1 & 2\, C\alpha C\gamma_3 & -2\, S\alpha S\gamma_3 - S\alpha C\gamma_1 \\ 0 & 0 & 3\, C\alpha \end{bmatrix} \begin{bmatrix} \dot{\gamma}_1 \\ \dot{\gamma}_3 \\ \dot{\alpha} \end{bmatrix}$$

$$+ J_P \dot{\beta} \begin{bmatrix} 0 & 3\, S\alpha & \left(-2\, C\alpha S\gamma_3 - C\alpha C\gamma_1\right) \\ -3\, S\alpha & 0 & \left(-C\alpha S\gamma_1\right) \\ \left(2\, C\alpha S\gamma_3 + C\alpha C\gamma_1\right) & \left(C\alpha S\gamma_1\right) & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 7)

Three Controller System (12-4-124)

Description of System: Same as four controller system except number 3 controller has failed.

Control Logic: Same as four controller system. $\alpha_1 = \alpha_2 = \alpha_4 \quad \gamma_1 = -\gamma_2$

Moment Contribution of Control System:

$$\sum M_{(12-4-124)}\Big]_A = J_P \dot{\beta} \begin{bmatrix} +2\, C\alpha C\gamma_1 & C\alpha S\gamma_4 & 2\, S\alpha S\gamma_1 + S\alpha C\gamma_4 \\ 0 & -C\alpha C\gamma_4 & S\alpha S\gamma_4 \\ 0 & 0 & 3\, C\alpha \end{bmatrix} \begin{bmatrix} \dot{\gamma}_1 \\ \dot{\gamma}_4 \\ \dot{\alpha} \end{bmatrix}$$

$$+ J_P \dot{\beta} \begin{bmatrix} 0 & 3\, S\alpha & C\alpha S\gamma_4 \\ -3\, S\alpha & 0 & \left(-2\, C\alpha S\gamma_1 - C\alpha C\gamma_4\right) \\ \left(-C\alpha S\gamma_4\right) & \left(2\, C\alpha S\gamma_1 + C\alpha C\gamma_4\right) & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 8)

Three Controller System (12-3-123)

Description of System: Same as four controller system except number 4 controller has failed.

Control Logic: Same as four controller system. $\alpha_1 = \alpha_2 = \alpha_3$ $\gamma_1 = -\gamma_2$

Moment Contribution of Control System:

$$\sum M_{(12\text{-}3\text{-}123)}\Big]_A = J_P \dot{\beta} \begin{bmatrix} 2\,C\alpha\, C\gamma_1 & -C\alpha\, S\gamma_3 & \left(2\,S\alpha\, S\gamma_1 - S\alpha\, C\gamma_3\right) \\ 0 & +C\alpha\, C\gamma_3 & \left(-S\alpha\, S\gamma_3\right) \\ 0 & 0 & 3\,C\alpha \end{bmatrix} \begin{bmatrix} \dot{\gamma}_1 \\ \dot{\gamma}_3 \\ \dot{\alpha} \end{bmatrix}$$

$$+ J_P \dot{\beta} \begin{bmatrix} 0 & 3\,S\alpha & -C\alpha\, S\gamma_3 \\ -3\,S\alpha & 0 & \left(-2\,C\alpha\, S\gamma_1 + C\gamma_3\, C\alpha\right) \\ C\alpha\, S\gamma_3 & \left(2\,C\alpha\, S\gamma_1 - C\alpha\, C\gamma_3\right) & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 9)

Four Controller System (12-34-1234)

Description of System: Four TDF Controllers are mounted in positions 1, 2, 3, and 4.

Control Logic:

$$\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = \begin{bmatrix} +1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \eta_x \\ \eta_y \\ \eta_z \end{bmatrix}$$

$+\dot{\gamma}_1$ gives roll to right.
$+\dot{\gamma}_3$ gives pitch nose down.
$+\dot{\alpha}$ gives yaw to left.

Moment Contribution of Control System:

$$\sum M_{(12-34-1234)}\Big]_A = 2\, J_P \dot{\beta} \begin{bmatrix} -C\alpha C\gamma_1 & 0 & -S\alpha S\gamma_1 \\ 0 & -C\alpha C\gamma_3 & +S\alpha S\gamma_3 \\ 0 & 0 & -2\, C\alpha \end{bmatrix} \begin{bmatrix} \dot{\eta}_x \\ \dot{\eta}_y \\ \dot{\eta}_z \end{bmatrix}$$

$$+ 2\, J_P \dot{\beta} \begin{bmatrix} 0 & 2\, S\alpha & -C\alpha S\gamma_3 \\ -2\, S\alpha & 0 & -C\alpha S\gamma_1 \\ +C\alpha S\gamma_3 & +C\alpha S\gamma_1 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 10)

The equation written in terms of input control variables is

$$\sum M_{(12-34-1234)}\Big]_A = 2\,J_P\,\dot{\beta}\begin{bmatrix} -C\alpha\, C\gamma_1 & 0 & S\alpha\, S\gamma_1 \\ 0 & C\alpha\, C\gamma_3 & -S\alpha\, S\gamma_3 \\ 0 & 0 & 2\,C\alpha \end{bmatrix}\begin{bmatrix} \dot{\gamma}_1 \\ \dot{\gamma}_3 \\ \dot{\alpha} \end{bmatrix}$$

$$+2\,J_P\,\dot{\beta}\begin{bmatrix} 0 & 2\,S\alpha & -C\alpha\, S\gamma_3 \\ -2\,S\alpha & 0 & -C\alpha\, S\gamma_1 \\ C\alpha\, S\gamma_3 & C\alpha\, S\gamma_1 & 0 \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 11)

Six Controller System (56-34-12)

Description of System: Six SDF Controllers are mounted in positions 1, 2, 3, 4, 5, and 6.

Control Logic:

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & -1 & 0 \\ +1 & 0 & 0 \\ +1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \eta_x \\ \eta_y \\ \eta_z \end{bmatrix}$$

$+\dot{\alpha}_5$ gives roll to right

$+\dot{\alpha}_3$ gives pitch nose down

$+\dot{\alpha}$ gives yaw left

Moment Contribution of Control System:

$$\sum M_{(56-34-12)}\Big]_A = 2 J_P \dot{\beta} \begin{bmatrix} -C\alpha_5 & 0 & 0 \\ 0 & -C\alpha_3 & 0 \\ 0 & 0 & -C\alpha_1 \end{bmatrix} \begin{bmatrix} \dot{\eta}_x \\ \dot{\eta}_y \\ \dot{\eta}_z \end{bmatrix}$$

$$+ 2 J_P \dot{\beta} \begin{bmatrix} 0 & +S\alpha_1 & -S\alpha_3 \\ -S\alpha_1 & 0 & -S\alpha_5 \\ +S\alpha_3 & +S\alpha_5 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 12)

The equation written in terms of control system input variables is

$$\sum M_{(56-34-12)}\Big]_A = 2\,J_P\,\dot{\beta}\begin{bmatrix} -C\alpha_5 & 0 & 0 \\ 0 & +C\alpha_3 & 0 \\ 0 & 0 & C\alpha_1 \end{bmatrix}\begin{bmatrix} \dot{\alpha}_5 \\ \dot{\alpha}_3 \\ \dot{\alpha}_1 \end{bmatrix}$$

$$+2\,J_P\,\dot{\beta}\begin{bmatrix} 0 & S\alpha_1 & -S\alpha_3 \\ -S\alpha_1 & 0 & -S\alpha_5 \\ +S\alpha_3 & +S\alpha_5 & 0 \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 13)

Three Controller Orthogonal System (5-3-1)

Description of System: System consists of SDF Controllers Numbers 1, 3, and 5.

Control Logic: None required since system has only three degrees of freedom and control system input matrix can be nearly diagonalized by arrangement.

$+\dot{\alpha}_5$ gives roll to right.
$+\dot{\alpha}_3$ gives pitch nose down.
$+\dot{\alpha}_1$ gives yaw to left.

Moment Contribution of Control System:

$$\sum M_{(5\text{-}3\text{-}1)}\Big]_A = J_P\dot{\beta}\begin{bmatrix} -C\alpha_5 & -S\alpha_3 & 0 \\ 0 & +C\alpha_3 & -S\alpha_1 \\ -S\alpha_5 & 0 & +C\alpha_1 \end{bmatrix}\begin{bmatrix} \dot{\alpha}_5 \\ \dot{\alpha}_3 \\ \dot{\alpha}_1 \end{bmatrix}$$

$$+J_P\dot{\beta}\begin{bmatrix} 0 & \left(S\alpha_1 + C\alpha_5\right) & \left(-C\alpha_1 - S\alpha_3\right) \\ \left(-S\alpha_1 - C\alpha_5\right) & 0 & \left(-S\alpha_5 + C\alpha_3\right) \\ \left(C\alpha_1 + S\alpha_3\right) & \left(S\alpha_5 - C\alpha_3\right) & 0 \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 14)

Three Controller Zero Momentum System (78-78-1)

Description of System: System consists of SDF Controllers Numbers 1, 7, and 8.

Control Logic: Although the system has only three degrees of freedom the control system input matrix cannot be written in diagonal form without control logic.

$$\begin{bmatrix} \alpha_1 \\ \alpha_7 \\ \alpha_8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ -1/2 & +1/2 & 0 \\ -1/2 & -1/2 & 0 \end{bmatrix} \begin{bmatrix} \eta_x \\ \eta_y \\ \eta_z \end{bmatrix}$$

Moment Contribution of Control System:

$$\sum M_{(78-78-1)}\Big]_A = J_P \dot{\beta} \begin{bmatrix} -1/2\left\{S\left(30^o+\alpha_7\right)+S\left(30^o-\alpha_8\right)\right\} & +1/2\left\{S\left(30^o+\alpha_7\right)-S\left(30^o-\alpha_8\right)\right\} & 0 \\ +1/2\left\{C\left(30^o+\alpha_7\right)-C\left(30^o-\alpha_8\right)\right\} & -1/2\left\{C\left(30^o+\alpha_7\right)+C\left(30-\alpha_8\right)\right\} & S\alpha_1 \\ 0 & 0 & -C\alpha_1 \end{bmatrix} \begin{bmatrix} \dot{\eta}_x \\ \dot{\eta}_y \\ \dot{\eta}_z \end{bmatrix}$$

$$+ J_P \dot{\beta} \begin{bmatrix} 0 & S\alpha_1 & \left\{S\left(30-\alpha_8\right)-C\alpha_1+S\left(30^o+\alpha_7\right)\right\} \\ -S\alpha_1 & 0 & \left\{C\left(30+\alpha_7\right)+C\left(30-\alpha_8\right)\right\} \\ \left\{-S\left(30-\alpha_8\right)+C\alpha_1-S\left(30^o+\alpha_7\right)\right\} & \left\{C\left(30+\alpha_7\right)-C\left(30-\alpha_8\right)\right\} & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 15)

Three Controller Zero Momentum System (178-78-178)

Description of System: System consists of TDF Controllers Numbers 1, 7, and 8.

Control Logic:

$$\begin{bmatrix} \gamma_1 \\ \gamma_7 \\ \gamma_8 \\ \alpha_1 \\ \alpha_7 \\ \alpha_8 \end{bmatrix} = \begin{bmatrix} +1 & 0 & 0 \\ -1/2 & +1/2 & 0 \\ -1/2 & -1/2 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \eta_x \\ \eta_y \\ \eta_z \end{bmatrix}$$

Moment Contribution of Control System:

$$\sum M_{(178-78-178)}\Big]_A =$$

$$+J_P\dot{\beta}\begin{bmatrix} -C\alpha\left\{+C\gamma_1+\frac{1}{2}C\left(\gamma_7-60\right)-\frac{1}{2}C\left(\gamma_8+60\right)\right\} & \frac{1}{2}C\alpha\left[C\left(\gamma_7-60\right)-C\left(\gamma_8+60\right)\right] & S\alpha\left[-S\gamma_1+S\left(\gamma_7-60\right)+S\left(\gamma_8+60\right)\right] \\ -C\alpha\left\{S\gamma_1+\frac{1}{2}S\left(\gamma_7-60\right)-\frac{1}{2}S\left(\gamma_8+60\right)\right\} & \frac{1}{2}C\alpha\left[S\left(\gamma_7-60\right)-S\left(\gamma_8+60\right)\right] & S\alpha\left[C\gamma_1-C\left(\gamma_7-60\right)-C\left(\gamma_8+60\right)\right] \\ 0 & 0 & -3\,C\alpha \end{bmatrix}\begin{bmatrix} \dot{\eta}_x \\ \dot{\eta}_y \\ \dot{\eta}_z \end{bmatrix}$$

$$+J_P\dot{\beta}\begin{bmatrix} 0 & 3\,S\alpha & -C\alpha\left[+C\gamma_1-C\left(\gamma_7-60\right)-C\left(\gamma_8+60\right)\right] \\ -3\,S\alpha & 0 & -C\alpha\left[S\gamma_1-S\left(\gamma_7-60\right)-S\left(\gamma_8-60\right)\right] \\ C\alpha\left[C\gamma_1-C\left(\gamma_7-60\right)-C\left(\gamma_8+60\right)\right] & C\alpha\left[S\gamma_1-S\left(\gamma_7-60\right)-S\left(\gamma_8-60\right)\right] & 0 \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 16)

Two Controller Yaw System (0-0-12)

$$\sum M_{(0-0-12)}\Big]_A = 2\, J_P\, \dot{\beta} \begin{bmatrix} q\, S\, \alpha_1 \\ -p\, S\, \alpha_1 \\ \dot{\alpha}_1\, C\, \alpha_1 \end{bmatrix}$$

Logic

$\alpha_1 = \alpha_2$

$+\dot{\alpha}_1$ gives yaw to left

(Eq. G. 2. 17)

Two Controller Pitch System (0-34-0)

$$\sum M_{(0-34-00)}\Big]_A = 2\, J_P\, \dot{\beta} \begin{bmatrix} -r\, S\, \alpha_3 \\ \dot{\alpha}_3\, C\alpha_3 \\ p\, S\alpha_3 \end{bmatrix}$$

Logic

$\alpha_3 = \alpha_4$

$+\dot{\alpha}_3$ gives pitch nose-down

(Eq. G. 2. 18)

Two Controller Roll System (56-0-0)

$$\sum M_{(56-0-0)}\Big]_A = 2\, J_P\, \dot{\beta} \begin{bmatrix} -\dot{\alpha}_5\, C\alpha_5 \\ -r\, S\alpha_5 \\ q\, S\alpha_5 \end{bmatrix}$$

Logic

$\alpha_5 = \alpha_6$

$+\dot{\alpha}_5$ = gives roll to right

(Eq. G. 2. 19)

MIT "Vertical Vee" (0-0-9, 10)

Description of System: System consists of two SDF Controllers.

Control Logic: None proposed since system is to be used as passive control system.

$$\sum M_{(0-0-9,10)}\Big]_A = J_P \dot{\beta} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -S\left(\alpha_0 - \alpha_9\right) & +S\left(\alpha_0 + \alpha_{10}\right) \\ 0 & +C\left(\alpha_0 - \alpha_9\right) & +C\left(\alpha_0 + \alpha_{10}\right) \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\alpha}_9 \\ \dot{\alpha}_{10} \end{bmatrix}$$

$$+ J_P \dot{\beta} \begin{bmatrix} 0 & -S\left(\alpha_0 - \alpha_9\right) + S\left(\alpha_0 + \alpha_{10}\right) & C\left(\alpha_0 - \alpha_9\right) + C\left(\alpha_0 - \alpha_{10}\right) \\ S\left(\alpha_0 - \alpha_9\right) - S\left(\alpha_0 + \alpha_{10}\right) & 0 & 0 \\ -C\left(\alpha_0 - \alpha\right) - C\left(\alpha_0 + \alpha_{10}\right) & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 20)

Four Controller Sun Pointing System (0-34-12)

Description of System: Six Controller system without Controllers Number 5 and 6.

$$\sum M_{(0\text{-}34\text{-}12)}\Big]_A = 2\,J_P\,\dot{\beta}\begin{bmatrix} 0 & 0 & 0 \\ 0 & -C\alpha_3 & 0 \\ 0 & 0 & -C\alpha_1 \end{bmatrix}\begin{bmatrix} 0 \\ \dot{\eta}_y \\ \dot{\eta}_z \end{bmatrix}$$

$$+2\,J_P\,\dot{\beta}\begin{bmatrix} 0 & S\alpha_1 & -S\alpha_3 \\ -S\alpha_1 & 0 & 0 \\ S\alpha_3 & 0 & 0 \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 21)

Logic

$$\begin{bmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \\ \dot{\alpha}_3 \\ \dot{\alpha}_4 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \\ 0 & 0 & +1 \end{bmatrix}\begin{bmatrix} 0 \\ \dot{\eta}_y \\ \dot{\eta}_z \end{bmatrix}$$

(Eq. G. 2. 22)

Three Orthogonal Inertia Wheels (Not a Gyro System)

Control Logic: None required because system has three degrees of freedom and control system input matrix can be diagonalized by inspection.

$$\sum M_{3\ \text{wheels}}\Big]_A = J_P \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{\beta}_1 \\ \ddot{\beta}_2 \\ \ddot{\beta}_3 \end{bmatrix}$$

$$+ J_P \begin{bmatrix} 0 & +\dot{\beta}_3 & -\dot{\beta}_2 \\ -\dot{\beta}_3 & 0 & +\dot{\beta}_1 \\ +\dot{\beta}_2 & -\dot{\beta}_1 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. G. 2. 23)

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BIOGRAPHICAL SKETCH

Noah C. New was born in Atlanta, Georgia on May 7, 1924 where he attended public schools. He entered the U. S. Navy on March 3, 1943 and became a naval aviator and a Marine Corps officer on March 3, 1947. As a reserve officer he entered the Georgia Institute of Technology and received the degree of Bachelor of Aeronautical Engineering in 1949 and the Master of Science in Aeronautical Engineering in 1950. He returned to active duty in the Marine Corps in October of 1950 and became a helicopter pilot instructor at Quantico, Virginia.

During 1952 he graduated in Class 8 of the Test Pilot Training School, Patuxent River, Maryland where he remained for two years as a helicopter project officer at the Flight Test Division of the Naval Air Test Center. The year 1954 was served in Japan as a member of the First Marine Aircraft Wing after which Noah C. New became the Navy Bureau of Aeronautics' Project Officer of a Sikorsky Marine Helicopter designated as the HR2S. He finished the Marine Corps Schools Junior Course in June 1959 after which he attended the Naval Postgraduate School, Monterey, California from which he received a Master of Science degree in June 1960.

He is married to the former Ellen Wiley of Atlanta, Georgia and has a daughter and two sons. He is a member of Sigma Nu, Sigma Gamma Tau, Tau Beta Pi, and Sigma Xi.